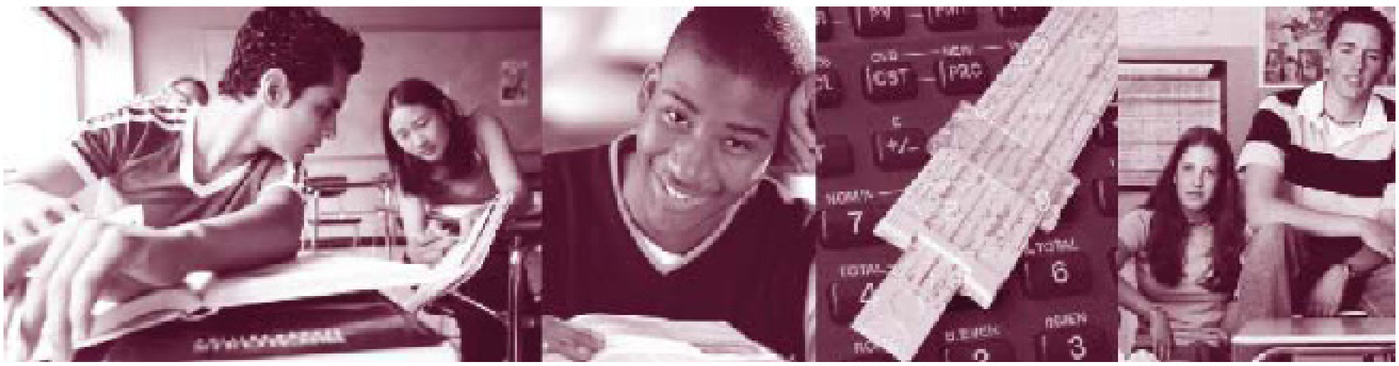
High School Content Expectations

Clarification Document



MATHEMATICS

Algebra

Topic A1.1: Construction, Interpretation, and Manipulation of Expressions

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Mathematics Clarification Document
Introduction

The Michigan Mathematics High School Content Expectations (HSCE) establish what every student is expected to know and be able to do by the end of high school and outline the parameters for receiving high school credit as recently mandated by the Michigan Merit Curriculum (MMC) legislation in the state of Michigan. In an effort to support mathematics teachers as they implement the Mathematic HSCE for all students, the Michigan Mathematics Leadership Academy (MMLA), a collaboration of the Michigan Mathematics and Science Center Network, the Michigan Council of Teachers of Education (MCTM) and the Michigan Department of Education, has developed this series of companion documents, with the goal of developing a common understanding of what the expectations mean. It is intended as a tool for mathematics educators and assessment developers to provide a clearer understanding of the breadth and depth of the HSCE.

Each document in the series represents one topic of the Mathematics HSCE and follows the same format:

- Information on the strand and standard where the topic is found. This information comes directly out of the HSCE document itself but may also include some further clarifying information.
- A list of the expectations in the topic including clarifying statements where needed.
- Background Information, Tools and Representations – a reference area that includes important formulas, properties, techniques and pedagogical tips for the topic in general.
- Assessable content intended as a technical guide for assessment developers, in addition to the information found in the rest of the document.
- A resource section that provides links to websites related to the topic.
- Clarifying Examples and Activities –include brief descriptions of classroom activities, assessment items and open-ended tasks all with the intent of providing further insights into the breadth and depth of each of the expectations. In many cases, suggested interventions for struggling students are provided.

Mathematics
Clarification for Topic A1.1: Construction, Interpretation, and Manipulation of Expressions

Strand: A-Algebra

In the middle grades, students see the progressive generalization of arithmetic to algebra. They learn symbolic manipulation skills and use them to solve equations. They study simple forms of elementary polynomial functions such as linear, quadratic, and power functions as represented by tables, graphs, symbols, and verbal descriptions.

In high school, students continue to develop their "symbol sense" by examining expressions, equations, and functions, and applying algebraic properties to solve equations. They construct a conceptual framework for analyzing any function and, using this framework, they revisit the functions they have studied before in greater depth. By the end of high school, their catalog of functions will encompass linear, quadratic, polynomial, rational, power, exponential, logarithmic, and trigonometric functions. They will be able to reason about functions and their properties and solve multi-step problems that involve both functions and equation-solving. Students will use deductive reasoning to justify algebraic processes as they solve equations and inequalities, as well as when transforming expressions.

This rich learning experience in Algebra will provide opportunities for students to understand both its structure and its applicability to solving real-world problems. Students will view algebra as a tool for analyzing and describing mathematical relationships, and for modeling problems that come from the workplace, the sciences, technology, engineering, and mathematics.

STANDARD: A1 – EXPRESSIONS, EQUATIONS, AND INEQUALITIES

Students recognize, construct, interpret, and evaluate expressions. They fluently transform symbolic expressions into equivalent forms. They determine appropriate techniques for solving each type of equation, inequality, or system of equations, apply the techniques correctly to solve, justify the steps in the solutions, and draw conclusions from the solutions. They know and apply common formulas.

> **Clarification:** A1 discusses the purely symbolic aspects of functions. This standard lays out the techniques that will be necessary to study any specific function. Functions are given by expressions; functions are then evaluated to find outputs. Finally, the task of finding inputs corresponding to a given output leads to the need to solve equations.
>
> Many of these expectations (e.g. A1.1.1, A1.2.1, A1.2.2, A1.2.3, and A1.2.8) are intended to frame the discussion throughout both algebra courses. So, for example, A1.1.1 means that students will evaluate quadratic expressions in Algebra I and rational expressions in Algebra II.

Topic A1.1 Construction, Interpretation, and Manipulation of Expressions

HSCE: A1.1.1 Give a verbal description of an expression that is presented in symbolic form, write an algebraic expression from a verbal description, and evaluate expressions given values of the variables.

Clarification: This expectation is to be developed throughout both algebra courses. The first phrase is about students developing the mathematical vocabulary needed for understanding algebra. They should be able to distinguish between powers, products, sums, etc. For example, describe 3^x as a constant raised to a variable power and x^3 as a variable raised to a constant power. Another example might be describing a quadratic function as the product of two linear functions.

HSCE: A1.1.2 Know the properties of exponents and roots, and apply them in algebraic expressions.

Clarification: e.g. $b^3 b^4 = b^{3+4} = b^7$

HSCE: A1.1.3 Factor algebraic expressions using, for example, greatest common factor, grouping, and the special product identities.

Clarification: see "Background Information, Tools, and Representations" below.

HSCE: A1.1.4 Add, subtract, multiply, and simplify polynomials and rational expressions.

Clarification: none

HSCE: A1.1.5 Divide a polynomial by a monomial.

Clarification: none

HSCE: A1.1.6 Transform exponential and logarithmic expressions into equivalent forms using the properties of exponents and logarithms including the inverse relationship between exponents and logarithms.

Clarification: e.g. $\log 100^3 = 3\log 100 = 6$

*__HSCE: A1.1.7__ Transform trigonometric expressions into equivalent forms using basic identities such as $\sin^2\theta + \cos^2\theta = 1$, $\tan\theta = \frac{\sin\theta}{\cos\theta}$, and $\tan^2\theta + 1 = \sec^2\theta$.

Clarification: This is a recommended expectation. It represents content that is desirable and valuable for all students, but attention to these items should not displace or dilute the curricular emphasis of any of the required expectations. Teachers are encouraged to incorporate the recommended expectations into their instruction when their students have a solid foundation and are ready for enrichment or advanced learning.

Background Information, Tools, and Representations

- Properties of Exponents

If a and b are real numbers and m and n are integers, the following properties of exponents can be used, in any order so long as orders of operation are considered.

Product of Powers: $a^m * a^n = a^{m+n}$
Power of a Power: $(a^m)^n = a^{mn}$
Power of a Product: $(ab)^m = a^m * b^m$
Negative Exponent: $a^{-m} = 1 / (a^m); a \neq 0$
Zero Exponent: $a^0 = 1; a \neq 0$
Quotient of Powers: $a^m / a^n = a^{m-n}; a \neq 0$
Powers of a Quotient: $(a / b)^m = a^m / b^m; b \neq 0$

- Properties of Logarithms

$\log_b 1 = 0$

$\log_b b = 1$

$\log_b b^x = x$

$\log_b(xy) = \log_b x + \log_b y$

$\log_b(x/y) = \log_b x - \log_b y$

$\log_b(x^y) = y \log_b x$

- Special Product Identities:

$(a + b)^2 = a^2 + 2ab + b^2;$
$(a - b)^2 = a^2 - 2ab + b^{2;}$
$(a - b)(a + b) = a^2 - b^2$

These patterns of multiplying binomials demonstrate “special products” of certain binomials. They also have been described as “factor patterns.”

- Use the x – intercepts of the linear components to aid in developing the concept of factoring
- Use Algebra tiles/blocks to help with factoring
- Use a graphing calculator to explore the inverse relationship between exponents and logarithms
- Logarithms (logs) are the "opposite" of exponentials, just as subtraction is the opposite of addition and division is the opposite of multiplication. Logs "undo" exponentials. Technically speaking, logs are the inverses of exponentials. Think of logs in terms of the relationship: $y = b^x$ which is equivalent to $\log_b y = x$, and is read "log-base-*b* of *y* equals *x*". "*b*" is called "the base of the logarithm", just as *b* is the base in the exponential expression $y = b^x$. Note that the base in both the exponential equation and the log equation (above) is "b", but that the *x* and *y* switch sides when you switch between the two equations. And, just as the base *b* in an exponential is always positive and not equal to 1, so also the base *b* for a logarithm is always positive and not equal to 1.
- The notation "ln(x)" means $\log_e(x)$ also called the natural log; the notation "log(x)" means $\log_{10}(x)$ also called the common log.

Assessable Content

- ❖ Each of these expectations may be assessed in conjunction with a specific function family (see Standard A3 topics).
- ❖ Students should also demonstrate knowledge of these expectations independent of a specific function family.
- ❖ Assessments should be limited to appropriate course function families and at the appropriate level of difficulty. For instance, don't use a polynomial of degree 5 when a polynomial of degree 3 will work just as well.

Resources

For more information on exponential functions:
http://www.purplemath.com/modules/expofcns.htm

For more information on inverse functions:
http://www.purplemath.com/modules/invrsfcn.htm

Translating Words into Symbols
http://www.learner.org/channel/workshops/algebra/workshop1

Students set up an equation that "matches" in numbers what the puzzle expresses in words.
http://www.lessontutor.com/eesA8.html

Tutorial on using Algebra tiles, the box method, and symbolic multiplication of binomials using the distributive law.
http://www.regentsprep.org/Regents/math/ALGEBRA/AV3/Smul_bin.htm

Explore Graphs and Factors, free download from activities exchange
http://education.ti.com/educationportal/activityexchange/activity_list.do?cid=us

Factoring and Solving Quadratic Binomials
http://education.ti.com/educationportal/plato/align

How to add/subtract polynomials
http://regentsprep.org/Regents/Math/polyadd/sp_add.htm

How to multiply polynomials
http://www.purplemath.com/modules/polymult.htm

How to divide polynomials
http://www.wtamu.edu/academic/anns/mps/math/mathlab/beg_algebra/beg_alg_tut30_divpoly.htm

Clarifying Examples and Activities

HSCE: A1.1.1

Give a verbal description of an expression that is presented in symbolic form, write an algebraic expression from a verbal description, and evaluate expressions given values of the variables.

Example 1

Here is a number trick:	Explain using algebraic expressions:
Pick a secret number. Double it. Add 9. Subtract 3. Divide by 2. Subtract your original number. Your answer will always be 3.	Let x = the number $2x$ $2x + 9$ $2x + 6$ $x + 3$ 3 That's why your answer is 3.
Choose a number Add five Double the result Subtract 4 Divide by 2 Subtract 3	x $x+5$ $2(x + 5)$ or $2x + 10$ $2x + 6$ $x+3$ x

Write the directions for doing these tricks backwards. Why does the trick work both ways? Explain using algebra expressions.

Extension:

Analyze this trick using algebraic expressions:

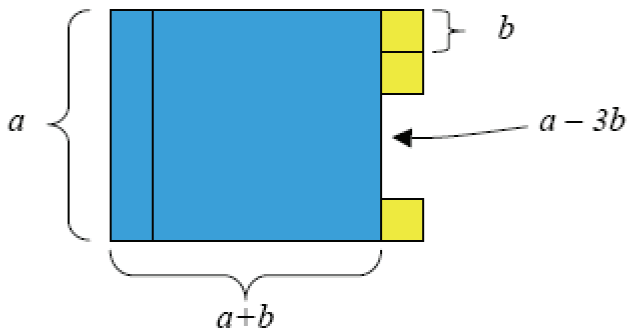
Ask somebody to think of a number and keep it secret.

- Double the secret number.
- Multiply by 5.
- Ask for the total.
- Whatever the total is, knock off the last digit.
- You have the secret number the person mentally chose at the start

Choose a number	x
Add five	$x + 5$
Double the result	$2(x + 5)$ or $2x + 10$
Subtract 4	$2x + 6$
Divide by 2	$x + 3$
Subtract 3	x

Example 2

What is the perimeter of this shape?



Answer: $a+2(a+b)+7b+(a-3b)$

(from NCTM Principles and Standards, pg. 282)

Example 3

A rectangular pool is to be surrounded by a ceramic tile border. The border will be one tile wide all around. Explain in words, with numbers or tables, visually, and with symbols the number of tiles that will be needed for pools of various lengths and widths.

Example 4

Written / Verbal	**Expression**	**Evaluation for x=2 and y=3**
The sum of three times x and the opposite of y	$3x-y$	3(2)-3
	$\frac{4}{y}+3x$	
The quantity x plus 3 divided by the quantity x minus 5		
x raised to the power of 3 times y		
The sine of 2x		
	4^x	

Suggestions for differentiation

- For the examples above, change expressions as needed.

Intervention

- Use algebra tiles or blocks to model the in Example 1 situations. Simplify expressions using the tiles. Examples:

  $2x - 3x$
  $3x - 2x + 7 - (x-3)$
  $2(x+3)$
  $2 - (x+3)$
  $2+(x+3)$

HSCE: A1.1.2

Know the properties of exponents and roots, and apply them in algebraic expressions.

Example 1

Change radical expressions (roots) to exponential expressions:

$$\sqrt{4} = 4^{1/2} = 2$$

$$\sqrt[3]{8} = 8^{1/3} = 2$$

Example 2

Have the students simplify expressions:

- Write each of the following in a simpler equivalent form without negative exponents

$$5x^0$$

$$\frac{10a^2}{5a}$$

$$7x^{-3}$$

- Write each of the following using radicals in simplified form

$$\sqrt{100+100}$$

$$\sqrt{\frac{44}{49}}$$

Example 3

If expanded, what is the greatest exponent of the variable *x* in the following expression?

$$x(x^2 - 4)\ (x + 3)(x^3 - 7x + 4)$$

Suggestions for differentiation

Intervention

Explore the pattern of exponents using the graphing calculator

HSCE: A1.1.3

Factor algebraic expressions using, for example, greatest common factor, grouping, and the special product identities

Example 1

Understand how to multiply/factor two binonials

This is the table version of the distributive property. Multiply (2*x*+3)(4*x*-5)
Use in reverse to factor.

times	**2x**	**3**
4x	$8x^2$	*12x*
-5	*-10x*	*-15*

This is also called the box method that appeared in the 15th century.

Example 2

If the expression $x^2 + kx + 9$ can be factored into the form $(x-b)^2$ or $(x+b)^2$, what are the possible values for *k*?
Answer:
6 or -6

Example 3

Have students use a graphing calculator to explore the relationship between the linear factors of a quadratic function and its zeros. This activity also allows students to recognize that the *x*-intercepts and axis of symmetry of a quadratic function can be determined from its linear factors.

HSCE: A1.1.4
Add, subtract, multiply, and simplify polynomials and rational expressions.

Example 1

How to subtract polynomials

$(4x^2 - 4) - (x^2 + 4x - 4)$

$(4x^2 - 4) + (-x^2 - 4x + 4)$ (Change the signs)

$4x^2 - 4 + -x^2 - 4x + 4$ (Clear the parenthesis)

$4x^2 - x^2 - 4x - 4 + 4$ (Combine the like terms)

$3x^2 - 4x$

Example 2

Multiply the following polynomials using an area model:

$$(x - 1)(2x^2 + 4x - 5)$$

	$2x^2$	$4x$	-5
x	$2x^3$	$4x^2$	$-5x$
-1	$-2x^2$	$-4x$	5

$$2x^3 + 4x^2 - 5x - 2x^2 - 4x + 5 = 2x^3 + 2x^2 - 9x + 5$$

Suggestions for differentiation

Intervention

A similar area model can be demonstrated with algebra tiles, either with algebra tile manipulatives or on with a computer applet.
http://nlvm.usu.edu/en/nav/category_g_4_t_2.html

HSCE: A1.1.5
Divide a polynomial by a monomial.

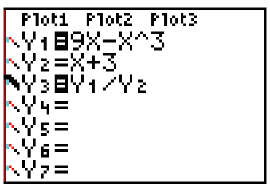
To divide a polynomial by a binomial, for example $\dfrac{9x - x^3}{x + 3}$:

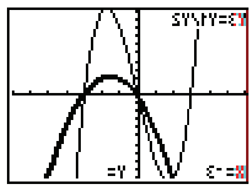
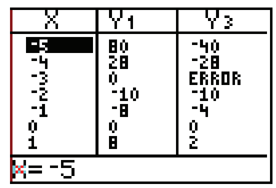
Example 1

$$\frac{9x - x^3}{x + 3}$$
$$\frac{x(9 - x^2)}{x + 3}$$
$$\frac{x(3 - x)(3 + x)}{x + 3}$$
$$x(3 - x)$$

Example 2

Functions Approach



X intercepts at (0, 0) and (3, 0) indicate a divisor of x(x-3)

Students can explore the possible quadratic equations that match the graph by using the Trace feature, Table feature, Calc feature, the zeros of the function, a regression equation, or other method.

Example 3

Long Division Approach

http://www.wtamu.edu/academic/anns/mps/math/mathlab/beg_algebra/beg_alg_tut30_divpoly.htm

HSCE: A1.1.6

Use the properties of exponents and logarithms, including the inverse relationship between exponents and logarithms, to transform exponential and logarithmic expressions into equivalent forms.

Example 1

Convert logs to exponents and vice versa

$y = \log_a x$ if and only if $x = a^y$

Base is a and exponent is y

If no base indicated, assume base 10

Logarithmic form	**Exponential form**
$\log_5 u = 2$	$5^2 = u$
$\log_b 8 = 3$	$b^3 = 8$
$r = \log_p q$	$p^r = q$
$x = \log 100$	$10^x = 100$

Example 2

Use properties of logs to convert logs to a form that is easier to solve or to use in a given situation

- $\log 100^3 = 3 \log 100 = 6$;

Suggestions for differentiation

Enhancement

❖ Introduce natural logs and *e*

Intervention

❖ Use base-10 and examples where students can easily see the connections

Michigan Department of Education
Betty Underwood, Interim Director
Office of School Improvement
www.michigan.gov/mde

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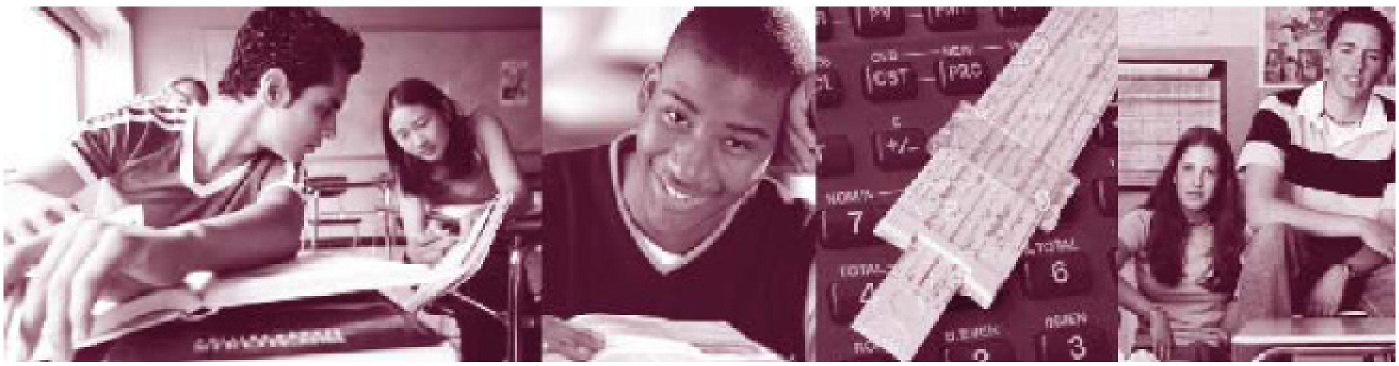

MATHEMATICS

Algebra

Topic A1.2: Solutions of Equations and Inequalities

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Michigan State Board of Education

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Mathematics
Clarification for Topic A1.2: Solutions of Equations and Inequalities

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Topic A1.2 Solutions of Equations and Inequalities

HSCE: A1.2.1 Write equations and inequalities with one or two variables to represent mathematical or applied situations, and solve.

Clarification:

- **Writing** of equations could require either one or two variables, e.g.
  (a) A shopper sees a coat that has been marked down 30% to $84. What was the original price? P- 0.3P =84; P=$120.
  (b) Sammy was paid $7.00 per hour to mow the lawn and $5.00 per hour to walk the dog. If he needs to make at least $35.00, how many hours should he do each job? 7x + 5y > 35
  (c) Write an equation whose solution is the x-value where the graph of $f(x) = \frac{1}{x^3} - 3$ crosses the x-axis.
- **Solving** equations and inequalities should be done in a single variable
- **Inequalities** should be kept to simple expressions, e.g., 4x + 5 < 9

HSCE: A1.2.2 Associate a given equation with a function whose zeros are the solutions of the equation.

Clarification: Understand and describe the relationships among the zeros of a function, the solutions of the associated equation, and the x-intercepts of the function's graph.

HSCE: A1.2.3 Solve (and justify steps in the solutions) linear and quadratic equations and inequalities, including systems of up to three linear equations with three unknowns; apply the quadratic formula appropriately.

Clarification: Linear systems may be solved using elimination or matrices. A specific symbolic method for solving quadratic equations is not prescribed. Students may use the vertex form instead of the quadratic formula; however, students should be familiar with the quadratic formula and how it is used. (See also A1.2.8)

HSCE: A1.2.4 Solve absolute value equations and inequalities, | x-3 | < 6, and justify steps in the solution.

Clarification: Absolute value functions are used to determine:

- distance from a to b on the number line |*a* - *b*| or| *b*- *a*|
- speed = |velocity|
- deviations from the mean: $|x - \bar{x}|$

HSCE: A1.2.5 Solve polynomial equations and equations involving rational expressions and justify steps in the solution.

Clarification: $-2x(x^2 + 4x+3) = 0$ is an example of a polynomial equation; x - (1/x + 6 = 3) is an example of an equation with a rational expression.

HSCE: A1.2.6 Solve power equations and equations including radical expressions, justify steps in the solution, and explain how extraneous solutions may arise.

Clarification: $(x+1)^3 = 8$ is a power equation. $\sqrt{(3x-7)} = 7$ is an equation with a radical expression.

HSCE: A1.2.7 Solve exponential and logarithmic equations and justify steps in the solution.

Clarification: The intent of the expectation is to solve the modeling equations such as $3.23(1.23)^t = 97$, $3(2^x) = 7$, and $2 \ln(x + 1) = 4$; not equations that require complicated manipulations such as $\log(x) - \log(x - 1)^3 = 2.123$.

HSCE: A1.2.8 Solve an equation involving several variables (with numerical or letter coefficients) for a designated variable, and justify steps in the solution.

Clarification: For example: A = P + Prt, solve for P; $\frac{1}{R_1} + \frac{1}{R_2} = 10$, solve for R_1

HSCE: A1.2.9 Know common formulas and apply appropriately in contextual situations.

Clarification: This expectation should be an overarching expectation throughout all the courses. Parts will be covered in each course with increasing complexity as students move through the curriculum.

HSCE: A1.2.10 Use special values of the inverse trigonometric functions to solve trigonometric equations over specific intervals.

Clarification: For example, $2 \sin x - 1 = 0$ for $0 \le x \le 2\pi$. The special values are those of the integer multiples of $\frac{\pi}{4}$ *and* $\frac{\pi}{6}$.

Background Information, Tools, and Representations

- ❖ Explore the relationship between quantitative variables that have a linear relationship. These include situations such as:
  - o Data represented in a linear graph
  - o Data represented in a table
  - o Data represented in a problem situation
- ❖ Common Formulas
  - o Point formulas: Given two points on a line $(x_1,,y_1)$*and*(x_2 , y_2)

Slope: $\frac{y_1 - y_2}{x_1 - x_2} = \frac{\Delta y}{\Delta x}$ *or rise/run*

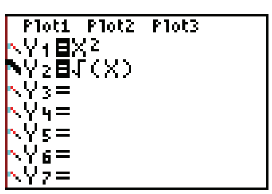
Midpoint: $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

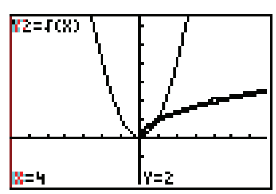
Distance: $d = \sqrt{\Delta x^2 + \Delta y^2}$ or $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$ *or rate∗time*

Quadratic x= $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ or x= $\frac{-b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$

Compound Interest: $A = P(1 + r)^n$

- ❖ Students need to understand the method of completing the square, which is the basis for the quadratic formula.
- ❖ Students should also understand the method of using the **distributive property** to multiply binomials (sometimes called the FOIL method):
  $(a + b)(c + d) = a(c + d) + b(c + d) = ac + ad + bc + bd$
- ❖ Students should understand that they could set two functions equal to each other and find the common solutions to them by exploring the table and graphs as well as solving algebraically.
- ❖ Students should have experiences solving equations with methods such as the following:
  1) Factoring/quadratic formula (when necessary)
  2) Graph the equations, either by hand or with a graphing calculator, determine x-intercepts, find linear factors
  3) Graphing the equation on a graphing calculator and then using the table feature to locate *x*-intercepts.
  4) Use a polynomial root finder on a graphing calculator or CAS system
- ❖ The square root function is defined to only return the non-negative root (a function can only return one unique number). The domain of the square root function is restricted to $x \geq 0$ to make this possible.





- An extraneous solution of an equation does not satisfy the original equation. To avoid incorrectly reporting extraneous solutions to equations, substitute that value into the original equation to see if it is a solution. It's especially important to do this when, for example, both sides of an equation were squared sometime during the solution method.

$$\sqrt{x+6}+2=-5$$
$$\sqrt{x+6}=-7$$
$$(\sqrt{x+6})^2=(-7)^2$$
$$x+6=49$$
$$x=43$$
$$\textit{Check}$$
$$\sqrt{43+6}+2=-5$$
$$9=-5$$

There is no solution. (Note Step 2 is problematic also)

- Other problem areas for students: Watch the order of operations.

```
-3²
             -9
(-3)²
              9
```

```
6-3²
             -3
6+-3²
             -3
6+(-3)²
             15
```

- Properties of Exponents and Logarithms

Exponents	**Logarithms**
(All laws apply for any positive *a*, *b*, *x*, and *y*.)	
$x = b^y$ is the same as $y = \log_b x$	
$b^0 = 1$	$\log_b 1 = 0$
$b^1 = b$	$\log_b b = 1$
$b^{(\log_b x)} = x$	$\log_b b^x = x$
$b^x\, b^y = b^{x+y}$	$\log_b(xy) = \log_b x + \log_b y$
$b^x \div b^y = b^{x-y}$	$\log_b(x/y) = \log_b x - \log_b y$
$(b^x)^y = b^{xy}$	$\log_b(x^y) = y \log_b x$

Assessable Content

- Each of these expectations may be assessed in conjunction with a specific function family (see Standard A3 topics).
- Students should also demonstrate knowledge of these expectations independent of a specific function family.
- Assessments should be limited to appropriate course function families and at the appropriate level of difficulty. For instance, don't use a polynomial of degree 5 when a polynomial of degree 3 will work just as well.
- **A1.2.9**: Students are expected to know the common formulas listed above without using a reference sheet. This item is **non-assessable** as a separate item but the common formulas may be used in items from other expectations, keeping in mind the level of the course and aligning with course appropriate function families.
- The justify portion of **A1.2.3** is **not assessable** in a multiple-choice format.

Resources

For a tutorial on absolute value
http://www.themathpage.com/alg/absolute-value.htm - equations

Apply Exponential Growth Problems
http://en.wikipedia.org/wiki/Exponential_growth#Examples_of_exponential_growth

Apply Exponential Decay Problems
$y = ab^x$ and 0<x<1
http://www.sosmath.com/algebra/logs/log5/log54/log54.html

Explore the effect of changing the parameters of *a* and *b* in $y = ab^x$
http://members.shaw.ca/ron.blond/TLE/ExpFcn.APPLET/index.html

Exponential Decay
http://www.wmich.edu/cpmp/unitsamples/c1u6/C1U6_439-448.pdf

Fractals and Fractal Dimension
http://www.shodor.org/interactivate/activities/FractalDimensions/?version=1.5.0_07&browser=safari&vendor=Apple_Computer,_Inc.
http://www.math.umass.edu/~mconnors/fractal/sierp/sierp.html

Movie Lines
This lesson allows students to apply their knowledge of linear equations and graphs in an authentic situation. Students plot data points corresponding to the cost of DVD rentals and interpret the results.
http://illuminations.nctm.org/LessonDetail.aspx?id=L629

Linear Relationships in Patterns
http://www.learner.org/channel/courses/learningmath/algebra/session5/part_a/index.html
http://www.learner.org/channel/courses/learningmath/algebra/session2/part_b/index.html

Proportional Reasoning and Direct Variation
Develop proportional reasoning skills by comparing quantities, looking at the relative ways numbers change, and thinking about proportional relationships in linear functions.
http://www.learner.org/channel/courses/learningmath/algebra/session4/index.html

Practice and Review Problems for common equations may be found at
http://www.themathpage.com/alg/equations.htm - four

What Goes Up Must Come Down
In this activity, students use the calculator to solve quadratic equations. They use the quadratic formula to determine the vertex and the x-intercepts of the graph of a quadratic function.
http://education.ti.com/educationportal/activityexchange/activity_detail.do?cid=us&activityid=5100

For Solving Quadratic equations, see TI graphing calculator programs at:
http://www.ticalc.org/pub/83plus/basic/math/quadratic/

Elimination Method for Solving Systems of Linear Equations
http://www.math.unc.edu/Faculty/mccombs/web/alg/classnotes/linsys/elim.html

Matrix Method for Solving Systems of Linear equations
http://algebralab.net/lessons/lesson.aspx?file=Algebra_matrix_systems.xml

Create and solve a system of linear equations in a real-world setting
http://illuminations.nctm.org/LessonDetail.aspx?id=L382

Solve a system of equations in a real-world setting.
http://illuminations.nctm.org/LessonDetail.aspx?id=L698

Clarifying Examples and Activities

HSCE: A1.2.1

Write equations and inequalities with one or two variables to represent mathematical or applied situations, and solve.

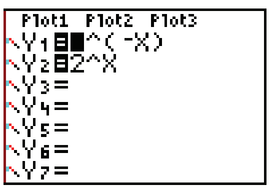
Example 1: Linear Equations

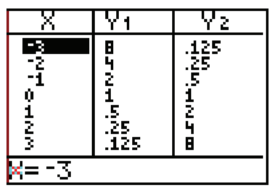
Explore the relationship between quantitative variables that have a **linear relationship**. These include situations such as:

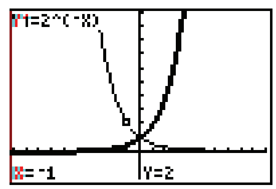
- Data represented in a linear graph
- Data represented in a table
- Data represented in a problem situation

Example 2: Exponential Equations

Use the graphing calculator to compare and contrast f(y) = 2^x with positive and negative numbers







In Algebra I, students solve exponential equations using tables and graphs on the graphing calculator, CAS, or spreadsheet technology. Solving exponential equations symbolically is addressed in Algebra II.

HSCE: A1.2.2

Associate a given equation with a function whose zeros are the solutions of the equation.

Example 1: Linear Equations

Write an equation whose solution is the *x*-value where the graph of $f(x)=\frac{1}{x^3}-3$ crosses the *x*-axis.

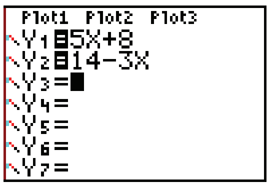
Example 2: Quadratic Equations

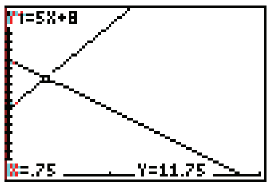
Find all of the integer values for c that allow the equation $y=x^2+6x+c$ to be factorable. You may use algebra tiles. Graph each equation. Write each of the equations as a product of linear factors (factored form). Identify the *x*-intercept(s) and *y*-intercept. Find the vertex of the parabola. Write a summary of your findings.

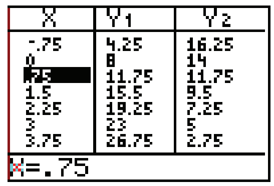
HSCE: A1.2.3
Solve linear and quadratic equations and inequalities, including systems of up to three linear equations with three unknowns. Justify steps in the solutions, and apply the quadratic formula appropriately.

Example 1
Use the multiple representations tools of the graphing calculator to explore the graphs of systems of linear equations and inequalities. Students may find solutions using graphs, (trace or intersect feature), tables, and symbolic reasoning. What are the patterns that suggest there is one solution, no solution, or many solutions?

Use a graphing calculator to solve the system of equations: $\begin{aligned} y &= 5x+8 \\ y &= 14-3x \end{aligned}$







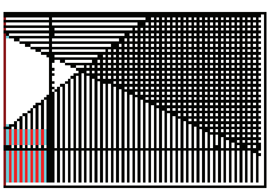
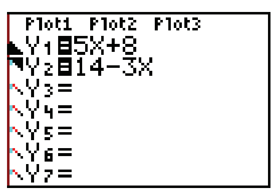
When x<.75, y1<y2. When x>.75, y1>y2. When x=.75, y1=y2.

Suggestions for differentiation

Enrichment

Solve the system of inequalities. $\begin{aligned} y &< 5x+8 \\ y &> 14-3x \end{aligned}$

Describe the region of the solution set.

HSCE: A1.2.4

Solve absolute value equations and inequalities.

Example 1

Compare and contrast the solutions of the three equations.

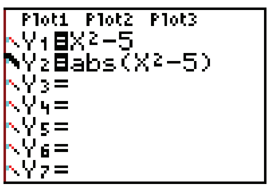
Solve $|x-5|=7$

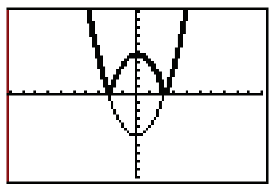
Solve $|x-5|\geq 7$

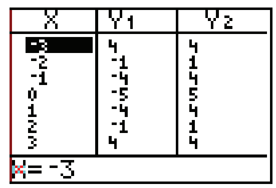
Solve $|x-5|\leq 7$

Suggestions for differentiation:

Sketch the graph of $y=|x^2-5|$ Explain the shape of the graph. What is the domain and range of the function?







Explore the shape of the absolute value of linear, quadratic, and cubic functions as parameters change.

Examples:

$f(x)=|x^2|+5$

$f(x)=|x^2+5|$

$f(x)=|x+5|$

HSCE: A1.2.5

Solve polynomial equations and equations involving rational expressions and justify steps in the solution.

Example 1

An example of a polynomial with a third power term

Suppose a square piece of tin measures 12 inches on each side. It is desired to make an open box from this material by cutting equal sized squares from the corners and then bending up the sides. Find a formula for the volume of the box as a function of the length x of the side of the square cut out of each corner.

The volume $V(x) = x(12-2x)^2$ is found by multiplying length•width•height. The formula for $V(x)$ may be expanded to $V(x) = 4x^3 - 48x^2 + 144x$, which is a sum of power functions with non-negative exponents. For which values of *x* does this formula represent the volume of the box?

Suggestions for differentiation

Enrichment

Application of Polynomials in Animation, real time imaging of motion
http://portal.acm.org/citation.cfm?id=646014.677626 - abstract

HSCE: A1.2.6

Solve power equations and equations including radical expressions, justify steps in the solution, and explain how extraneous solutions may arise.

Example 1

Multiple Choice Assessment Items

Solve for x: $\sqrt{x-8}-8=2$

A. 108
B. -68
C. 2
D. No Solution

Answer: A. 108

Solve for x: $(x+4)^2=-16$

A. 8
B. -8
C. 0
D. No Solution

Answer: D. No Solution

Solve for x: $\sqrt{x+6}+2=-5$

A. 43
B. 27
C. -9
D. No Solution

Answer: D. No Solution

Solve for x: $(x-2)^3=27$

A. 11
B. 5
C. 7
D. No Solution

Answer: B. 5

Suggestions for differentiation

Intervention

- **Stacking Squares**

This lesson prompts students to explore ways of arranging squares to represent equivalences involving square- and cube – roots. Students' explanations and representations (with their various ways of finding these roots) form the basis for further work with radicals.

http://illuminations.nctm.org/LessonDetail.aspx?id=L622

HSCE: A1.2.7

Solve exponential and logarithmic equations, and justify steps in the solution.

Example 1

Using the basic properties of logs, solve the following equations.

Solve for x:

$$3^x = 8$$

$$\log 3^x = \log 8$$

$$x * \log 3 = \log 8$$

$$x = \frac{\log 8}{\log 3}$$

$$x \approx 1.8929$$

Example 2

Solve for x:

$$3.23(1.23)^x = 97$$

$$\frac{3.23(1.23)^x}{3.23} = \frac{97}{3.23}$$

$$(1.23)^x = \frac{97}{3.23}$$

$$\log(1.23)^x = \log\frac{97}{3.23}$$

$$x\log(1.23) = \log\frac{97}{3.23}$$

$$x = \log\frac{97}{3.23} * \frac{1}{\log(1.23}$$

$$x \approx 16.4$$

Suggestions for differentiation:

Intervention

Practice solving log equations

http://www.purplemath.com/modules/solvelog.htm

HSCE: A1.2.8

Solve an equation involving several variables (with numerical or letter coefficients) for a designated variable. Justify steps in the solution.

Example 1

$$A = P + \Pr t$$

$$A = P(1 + rt)$$

$$\frac{A}{1 + rt} = P$$

Example 2

Rewrite common area and perimeter rules.

$$A = L * W \Leftrightarrow L = \frac{A}{W}$$

$$A = \pi r^2 h \Leftrightarrow h = \frac{A}{\pi r^2}$$

$$P = 2L + 2W \Leftrightarrow P = 2(L + W)$$

Example 3

Rewrite the Pythagorem formula.

$$x^2 + y^2 = d^2$$

$$d = \sqrt{x^2 + y^2}$$

$$x = \sqrt{d^2 - y^2}$$

$$y = \sqrt{d^2 - x^2}$$

Example 4

Find the slope and *y*-intercept of 7x+3y=15.

$$7x + 3y = 15$$

$$3y = 15 - 7x$$

$$y = \frac{15 - 7x}{3}$$

$$y = \frac{15}{3} - \frac{7x}{3}$$

$$y = 5 - 2\frac{1}{3}x$$

Example 5

For quadratic equations, solve for *x* by completing the square (may model with algebra tiles).

$$x^2 + 4x - 2 = 0$$
$$x^2 + 4x + 4 - 4 - 2 = 0$$
$$(x+2)^2 - 6 = 0$$
$$(x+2)^2 = 6$$
$$x + 2 = \pm\sqrt{6}$$
$$x = \pm\sqrt{6} - 2$$

$$2x^2 + 8x - 4 = 0$$
$$\frac{2x^2 + 8x - 4}{2} - 0$$
$$x^2 + 4x - 2 = 0$$
$$x^2 + 4x + 4 - 4 - 2 = 0$$
$$(x+2)^2 - 6 = 0$$
$$(x+2)^2 = 6$$
$$x + 2 = \pm\sqrt{6}$$
$$x = \pm\sqrt{6} - 2$$

HSCE: A1.2.9
Know common formulas, and apply appropriately in contextual situations.

Example 1
Generating Spreadsheet Formulas:

Explain how the Fill Down function generates the recursive formula for the each cell.

Example: A3=A2+1, A4=A3+1

Week	**Amount**
0	100
1	107
2	114
3	121
4	128
5	135

List functions on the graphing calculator are useful to generate formulas and rules to find the patterns in data stored in lists.

	A	B
1	Week	Amount
2	0	100
3	=A2+1	=B2+7
4	=A3+1	=B3+7
5	=A4+1	=B4+7
6	=A5+1	=B5+7
7	=A6+1	=B6+7

Suggestions for differentiation

Enrichment
The History behind the Quadratic Formula
http://www.bbc.co.uk/dna/h2g2/A2982567

Twenty ways to solve a quadratic equation; many historical.
www.pballew.net/quadsol.doc

Intervention

- ❖ **Understanding how to multiply two binonials:**

This is a form of the distributive law . Multiply (2x+3)(4x-5)

Multiply	**2x**	**3**
4x	***8x²***	***12x***
-5	***-10x***	***-15***

This is the box method that appeared in the 15th century.

- ❖ **Solve by completing the square.**

Model with algebra tiles if needed or use the box method from above.

$x^2 + 4x - 21 = 0$
$x^2 + 4x + 4 - 4 - 21 = 0$
$(x+2)^2 - 25 = 0$
$(x+2)^2 = 25$
$x + 2 = \pm 5$
$x = \pm 5 - 2$

$x^2 + 4x - 2 = 0$
$x^2 + 4x + 4 - 4 - 2 = 0$
$(x+2)^2 - 6 = 0$
$(x+2)^2 = 6$
$x + 2 = \pm\sqrt{6}$
$x = \pm\sqrt{6} - 2$

HSCE: A1.2.10

Use special values of the inverse trigonometric functions to solve trigonometric equations over specific intervals.

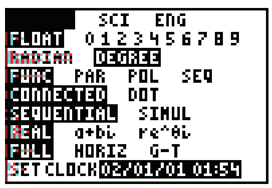
Example 1

Introduce the unit circle as a circle with radius 1 centered at the origin. Define sine as the *y*-value and cosine as the *x*-value of the position coordinates on the circle.

Example 2

Use the graphs and tables of the trig functions to find the values of angles of multiples of 45 degrees or $\frac{\pi}{2}$ radian. Relate these values to the corresponding coordinate positions on the unit circle.

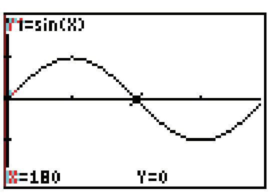
(*Set the mode of the calculator to radians or degrees)*.

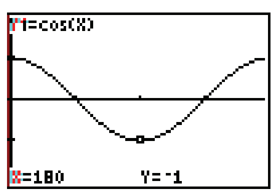


Example 3

Solve $y = \sin^{-1}\theta$ and $y = \cos^{-1}\theta$ over the domain $0^\circ < \theta < 360^\circ$.
Using the values in quadrant 1, apply symmetry patterns to find the exact values for $\theta = 0°, 45°, 90°, 135°, 180°, 225°, 270°, 315°$ and $360°$. Draw the unit circle. Label the *x* and *y* coordinates of each angle on the unit circle.

Relate to the coordinates on the sine and cosine graph.

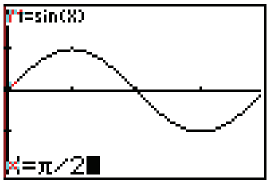


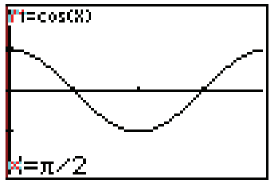


Example 4

Solve $y = \sin^{-1}\theta$ and $y = \cos^{-1}\theta$ over the domain $0^\circ < \theta < 2\pi$ **radians**. Using the values in quadrant 1, apply symmetry patterns to find the exact values for
$\theta = .\ 0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi, \frac{5\pi}{4}, \frac{3\pi}{2}, \frac{7\pi}{4}, 2\pi$.
Draw the unit circle. Label the *x* and *y* coordinates of each angle on the unit circle.

Relate to the coordinates on the sine and cosine graph





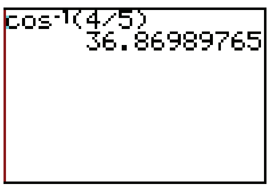
Use the same approach to solve $y = \tan^{-1}\theta$. The solutions will generate the slope of the hypotenuse of the triangles on the unit circle.

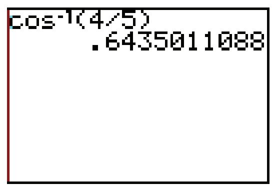
Example 5

Solve the following for $0^{\circ} < \theta < 90^{\circ}$ or $0 < \theta < \dfrac{\pi}{2}$

$\cos^{-1}\theta = \dfrac{4}{5}$ (degrees)

$\cos^{-1}\theta = \dfrac{4}{5}$ (radians)





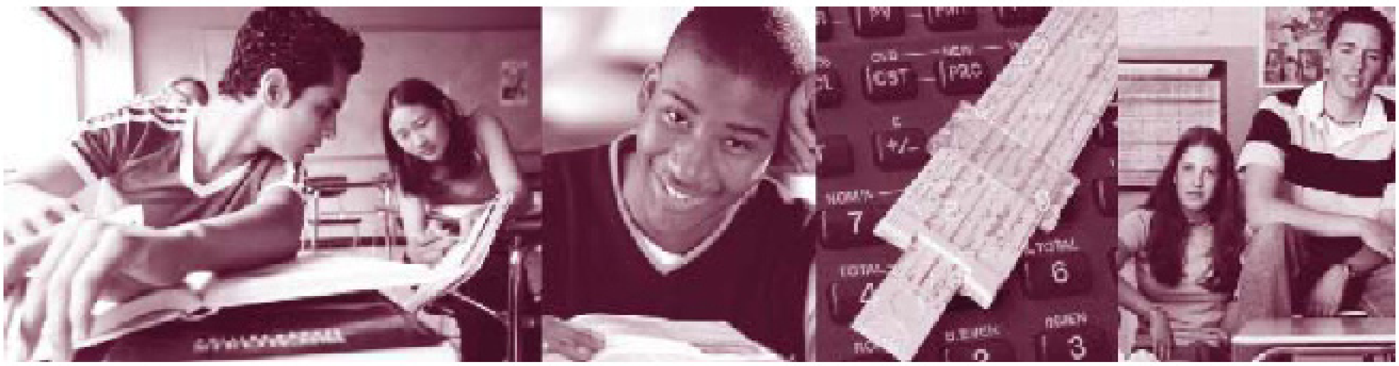
Solution: $\theta \approx 36.9^{\circ}$

Solution: $\theta \approx 0.64 rad$

Michigan Department of Education
Betty Underwood, Interim Director
Office of School Improvement
www.michigan.gov/mde

High School Content Expectations

Clarification Document



MATHEMATICS

Algebra
Topic A2.1: Definitions, Representations, and Attributes of Functions

Michigan State Board of Education

Mathematics Clarification Document
Introduction

The Michigan Mathematics High School Content Expectations (HSCE) establish what every student is expected to know and be able to do by the end of high school and outline the parameters for receiving high school credit as recently mandated by the Michigan Merit Curriculum (MMC) legislation in the state of Michigan. In an effort to support mathematics teachers as they implement the Mathematic HSCE for all students, the Michigan Mathematics Leadership Academy (MMLA), a collaboration of the Michigan Mathematics and Science Center Network, the Michigan Council of Teachers of Education (MCTM) and the Michigan Department of Education, has developed this series of companion documents, with the goal of developing a common understanding of what the expectations mean. It is intended as a tool for mathematics educators and assessment developers to provide a clearer understanding of the breadth and depth of the HSCE.

Each document in the series represents one topic of the Mathematics HSCE and follows the same format:

- Information on the strand and standard where the topic is found. This information comes directly out of the HSCE document itself but may also include some further clarifying information.
- A list of the expectations in the topic including clarifying statements where needed.
- Background Information, Tools and Representations – a reference area that includes important formulas, properties, techniques and pedagogical tips for the topic in general.
- Assessable content intended as a technical guide for assessment developers, in addition to the information found in the rest of the document.
- A resource section that provides links to websites related to the topic.
- Clarifying Examples and Activities –include brief descriptions of classroom activities, assessment items and open-ended tasks all with the intent of providing further insights into the breadth and depth of each of the expectations. In many cases, suggested interventions for struggling students are provided.

Mathematics
Clarification for Topic A2.1: Definitions, Representations, and Attributes of Functions

Strand: A-Algebra

In the middle grades, students see the progressive generalization of arithmetic to algebra. They learn symbolic manipulation skills and use them to solve equations. They study simple forms of elementary polynomial functions such as linear, quadratic, and power functions as represented by tables, graphs, symbols, and verbal descriptions.

In high school, students continue to develop their "symbol sense" by examining expressions, equations, and functions, and applying algebraic properties to solve equations. They construct a conceptual framework for analyzing any function and, using this framework, they revisit the functions they have studied before in greater depth. By the end of high school, their catalog of functions will encompass linear, quadratic, polynomial, rational, power, exponential, logarithmic, and trigonometric functions. They will be able to reason about functions and their properties and solve multi-step problems that involve both functions and equation-solving. Students will use deductive reasoning to justify algebraic processes as they solve equations and inequalities, as well as when transforming expressions.

This rich learning experience in Algebra will provide opportunities for students to understand both its structure and its applicability to solving real-world problems. Students will view algebra as a tool for analyzing and describing mathematical relationships, and for modeling problems that come from the workplace, the sciences, technology, engineering, and mathematics.

STANDARD: A2 – FUNCTIONS

Students understand functions, their representations, and their attributes. They perform transformations, combine and compose functions, and find inverses. Students classify functions and know the characteristics of each family. They work with functions with real coefficients fluently.

Students construct or select a function to model a real-world situation in order to solve applied problems. They draw on their knowledge of families of functions to do so.

Clarification

This standard focuses on the foundations and application of functions. It is the anchor for the other two algebra standards. All work with expressions, equations (A1) and specific function families (A3) needs to support the bigger understanding of the usefulness of functions. All of the topics in this standard are intended to frame the discussion about function families throughout both algebra courses.

Topic A2.1: Definitions, Representations, and Attributes of Functions

HSCE: A2.1.1 Determine whether a relationship (given in contextual, symbolic, tabular, or graphical form) is a function; and identify its domain and range.

Clarification: Students should be able to determine the function family from any of the representations (rule, graph, table, or context).

HSCE: A2.1.2 Read, interpret, and use function notation, and evaluate a function at a value in its domain.

Clarification: Let C(q) = 200 + 0.1q be the cost in dollars of manufacturing q items. Find and interpret C(100). Solve C(q) = 300. What does your answer mean in terms of items and dollars?

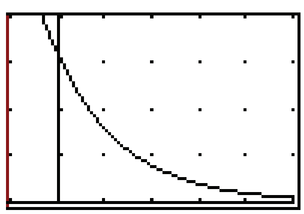
HSCE: A2.1.3 Represent functions in symbols, graphs, tables, or words, and translate among representations.

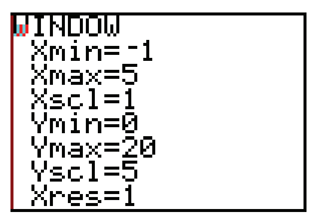
Clarification: Symbolic: $f(x) = 16(\frac{1}{2})^x$

Table:

x	0	1	2
y	16	8	4

Graphical:





Contextual (verbal): Due to a cleanup campaign the number of tons of a pollutant P is being reduced by 50% each year. Originally there were 16 tons of pollutant, and the cleanup will end when there is only 1 ton remaining.

HSCE: A2.1.4 Recognize that functions may be defined by different expressions over different intervals of their domains; such functions are piecewise-defined.

Clarification: e.g., absolute value and greatest integer function (floor function).

HSCE: A2.1.5 Recognize that functions may be defined recursively, and compute values of and graph simple recursively defined functions.

Clarification: none

HSCE: A2.1.6 Identify the zeros of a function and the intervals where the values of a function are positive or negative, and describe the behavior of a function, as x approaches positive or negative infinity, given the symbolic and graphical representations.

Clarification: Students should be able to find the zeros in a variety of ways, symbolically when appropriate, graphically, or with a table. This is not an expectation about limits; it only means having a sense of the overall shape of a function. So, positive infinity means to consider large positive values for x and negative infinity means to consider large negative values of x. Some teachers may be familiar with this as "end behavior." For example:

- The function $f(x) = x^2 - 9$ has zeros at x = 3 and x = -3. It is negative between the two zeros and otherwise positive. For large x values (positive or negative) the function values are large and positive.
- The graph of $f(x) = \frac{1}{x^2}$ approaches the x-axis for large values of x.

HSCE: A2.1.7 Identify and interpret the key features of a function from its graph or its formula(e).

Clarification: Key features may include slope, intercept(s), asymptote(s), maximum and minimum value(s), symmetry, average rate of change over an interval, and periodicity. Each type of function may have different characteristic features, for example, slope for linear and periodicity for trigonometric functions.

Background Information, Tools, and Representations

- ❖ Graphing calculators should be used to allow students to view different representations (table, graph or rule) of the same function.
- ❖ A function's **domain and range** depend on the form in which it is presented. For instance, the symbolic exponential function $f(x) = 16(\frac{1}{2})^x$ has a domain that includes all real x and a range of all positive y. A verbal representation of this function could be: "Due to a cleanup campaign the number of tons of a pollutant P is being reduced by 50% each year. Originally there were 16 tons of pollutant, and the cleanup will end when there is only 1 ton remaining." This is more restrictive. In 4 years there will be one ton of pollutant left, so the domain is $0 < t < 4$ and the range is $1 < P < 16$.
- ❖ Students should recognize that **asymptotes** exist and be able to talk about why they exist. In Algebra 1 they should not be required to write the equation of asymptotes.
- ❖ A recursive function contains two important components:
  - A recursion formula that tells how any term of a sequence relates to the preceding terms (e.g., $x_n = x_{n-1} + 2$, or Next = Now + 2); and an initial condition that gives the starting point (e.g., $x_1 = 1$ or Start = 1).
  - The initial condition is necessary to ensure a uniquely defined sequence. The example above gives the sequence of odd numbers 1, 3, 5, 7... . However, if the initial condition was modified to $x_1 = 2$ or Start = 2, the recursive function would give the sequence of even numbers 2, 4, 6, 8... .

Assessable Content

- ❖ Each of these expectations may be assessed in conjunction with a specific function family (see Standard A3 topics).
- ❖ Students should also demonstrate knowledge of these expectations independent of a specific function family.
- ❖ Assessments should be limited to appropriate course function families and at the appropriate level of difficulty. For instance, don't use a polynomial of degree 5 when a polynomial of degree 3 will work just as well.

Resources

- ❖ Now-Next Recursive Rules
  http://www.learner.org/channel/workshops/algebra/workshop5/index2.html - 1
- ❖ Floor and ceiling functions
  The names "floor" and "ceiling" and the corresponding notations were introduced by Kenneth E. Iverson in 1962 (http://en.wikipedia.org/wiki/Floor_function).
  Excel Spreadsheet Functions (floor and ceiling):
  http://www.techonthenet.com/excel/formulas/floor.php
  http://www.techonthenet.com/excel/formulas/ceiling.php

Clarifying Examples and Activities

HSCE: A2.1.1

Determine whether a relationship (given in contextual, symbolic, tabular, or graphical form) is a function; and identify its domain and range.

Example 1

Snow begins falling *steadily* at time $t = 0$. The ground already has some snow on it. At time $t = 2$, it is covered to a depth of 8 inches. Four hours later (at time $t = 6$), the depth is 12 inches.

- Make a table of values for $t = 0$ through 10 hours in increments of .5 hours.
- Graph the data: time (hours), depth (feet). What appears to be the relationship between the two quantities?
- Find a possible equation for d, the depth of snow coverage (in inches), as a function of t, the amount of time elapsed (in hours).
- Explain how the table, graph, and function model the relationship between (t,d).
- Write a weather report about the snowfall. Include the domain and range as part of your report. Make a prediction about the depth of the snow to expect in 24 hours if snowfall continues at the current rate.

HSCE: A2.1.2

Read, interpret, and use function notation and evaluate a function at a value in its domain.

Example 1

The children in a family contribute equally for a gift to their mother. Each child contributes $F(c) = c/5$, where c is the total cost of the gift.

The individual cost is a function of the total cost of the gift.

How many children are there?

If the gift costs $200, how much would each child need to contribute?

Find the value of c if each child contributes $50.

Find $F(100)$. Explain what $F(100)$ describes about the situation.

Example 2:

Some relatives in a family decide to contribute equally for a Christmas donation for a needy family. Each relative contributes $F(n) = 300/n$, where n is the total number of relatives.

The individual cost is a function of the number of relatives willing to donate for the gift.

How much does the Christmas basket cost?

Find the cost per person if 15 relatives decide to donate?

Find $F(10)$. Explain what $F(10)$ describes about the situation.

Write a problem that can be answered by the function.

HSCE: A2.1.3
Represent functions in symbols, graphs, tables, or words, and translate among representations.

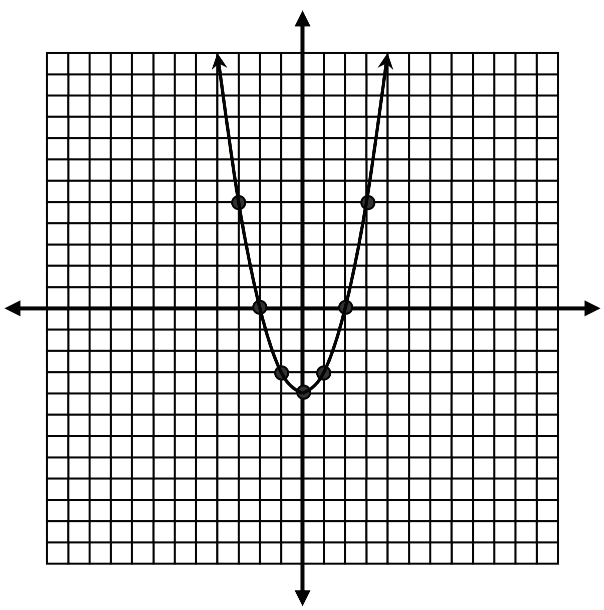
Example 1

x	y
8	1
4	2
2	4
0.5	16

Write an equation for the set of data represented in the table above. Graph the data. Verify that the equation fits the data set.

Example 2
The NCAA Basketball Tournament is single elimination (once a team loses, it is out of the tournament) and starts with 64 teams. Create a function table and graph for this situation.

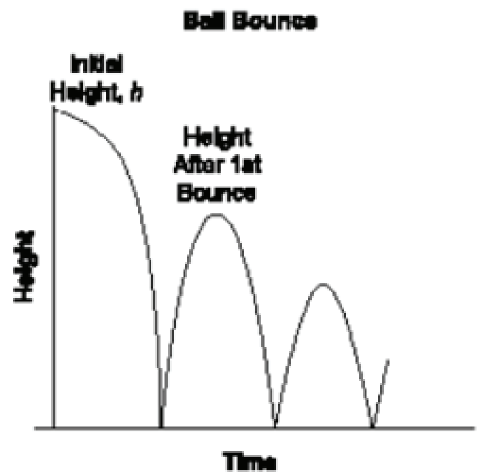
Example 3



Find the equation whose graph is pictured above. Verify that the equation describes the graph.

Example 4

When a ball is dropped from height *h*, it bounces to 2/3 of its initial height. When the ball bounces a second time, it reaches 2/3 the height after the first bounce, and so on. Which expression shows the height of the ball after its *n*th bounce?



A $\left(\frac{2}{3}\right)^n h$

B $n\left(\frac{2}{3}h\right)$

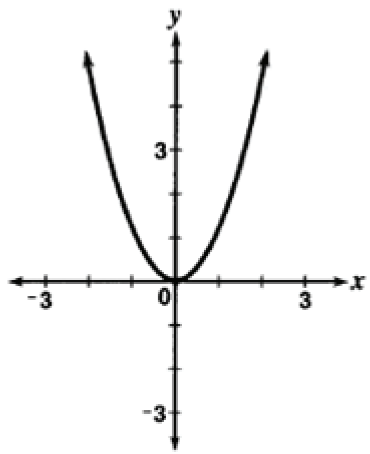
C $\left(\frac{2}{3}h\right)^n$

D $\frac{2}{3}h^n$

Source: 2004 MEAP Released items, High School Test

Example 5

Look at the graph below. Which table BEST represents the graph?



A

x	y
–2	4
–1	1
0	0
1	1

B

x	y
–2	4
–1	1
0	0
1	–1

C

x	y
–2	–4
–1	1
0	0
1	1

D

x	y
–2	–4
–1	1
0	0
1	–1

Source: MEAP Released Items, High School Test, 2000

HSCE: A2.1.4
Recognize that functions may be defined by different expressions over different intervals of their domains; such functions are piecewise-defined

Example 1

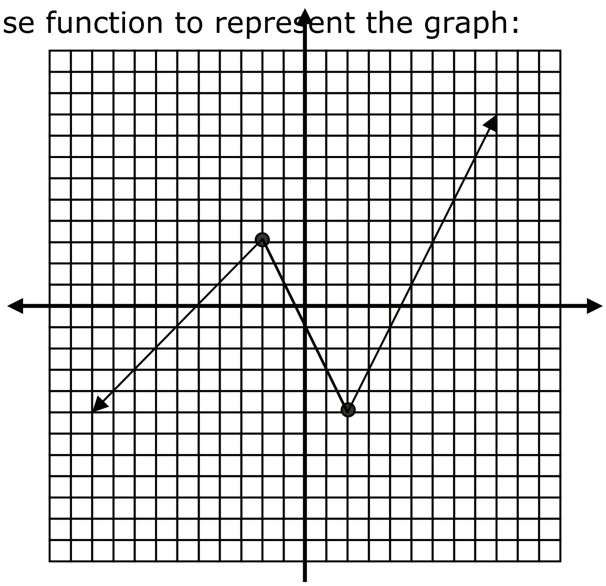
- A taxi charges \$1.50 for the first ¼ mile and \$0.25 for each additional ¼ mile (rounded up to the nearest ¼).
- Make a table showing the cost of a trip as a function of its length.
- Graph the cost function.

Example 2

Graph the function $f(x) = |x|$. (Absolute value function)

Example 3

Write a piecewise function to represent the graph:



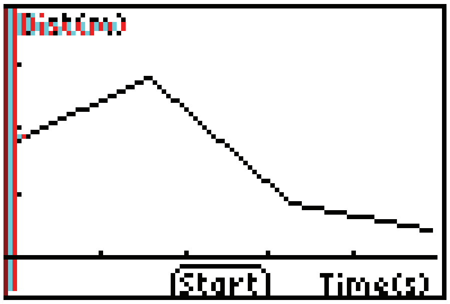
The function below is one possibility. One could change the first inequality to "x < -2" and the second to "-2 ≤ x ≤ 2"and leave the third alone. There is also one other possibility.

$$f(x) = \begin{cases} x+5 & \text{if } x \le -2 \\ -2x-1 & \text{if } -2 < x \le 2 \\ 2x-9 & \text{if } x > 2 \end{cases}$$

Example 4
Graph the function $f(x) = [x]$ (the greatest integer function; also called the floor function $\lfloor x \rfloor$)
Graph the function $f(x) = \lceil x \rceil$ (also called the ceiling function).
(See Resources for more information)

Example 5

CBR Ranger Walk the Graph Activity
Write a possible piecewise function for the (time, position) graph. Set the window domain and range to match the graph window. Write a summary of how to walk the graph using info from the equations.



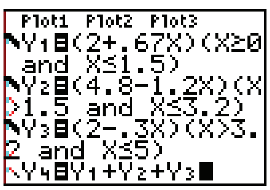
y1=____________________

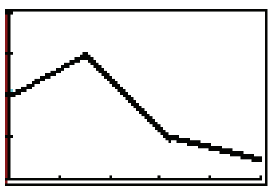
y2=____________________

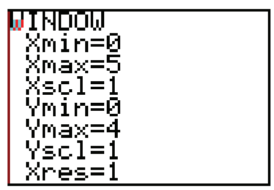
y3= ____________________

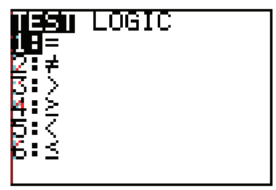
y4= y1+y2+y3= ____________________

Possible Answer: allow for variation









Limit the domain of a function by using Boolean expressions. These are statements using inequality and equality tests that have a value of 1 when true and 0 when false. To find these on the TI-83/84, press 2[nd] Math. The functions in Y1, Y2, and Y3 are multiplied by a Boolean expression. Multiplication by 1 yields the function value f(x) for all *x*'s in the intervals. Multiplication by 0 yields a 0 for all f(x) values for all *x*'s in the interval. The functions Y1, Y2, and Y3 may also be divided by the Boolean expression. Division by 1 will allow the function to compute in the interval, division by zero will not allow the function to graph (division by zero not allowed) in the interval.

Website: How to graph piecewise functions on the TI-83/84
http://www.prenhall.com/esm/app/graphing/ti83/Graphing/Special_graphs/piece_wise/piece_wise.html

HSCE: A2.1.5

Recognize that functions may be defined recursively, and compute values of and graph simple recursively defined functions.

Example 1

Describe Linear and Exponential data patterns recursively

Linear ($a+bx$) has a constant rate of change: next = now +b

Exponential (ab^x) has a constant multiplier: next = now *b

Example 2

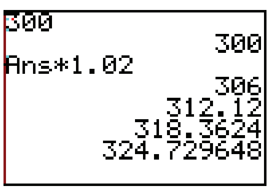
An initial population of 300 people grows at the rate of 2% per year. Use a recursive routine to generate a sequence of population values.

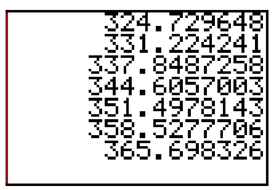
$$P_0 = 300$$
$$P_1 = P_0 * 1.02$$
$$P_2 = P_1 * 1.02$$
$$P_3 = P_2 * 1.02.......$$
$$P_{10} = P_9 * 1.02$$

Next year=Previous year *1.02 starting with an initial population of P_0

Example 3

The recursive rule above may be used to generate a table of values using the ANS feature of the graphing calculator. Start with a population of 300.





Answer: 366 people

The recursive routine is used to find the explicit form of an exponential function.

$$P_{10} = P_0 * 1.02 * 1.02 * 1.02 * 1.02 * 1.02 * 1.02 * 1.02 * 1.02 * 1.02 * 1.02$$
$$P_{10} = P_0 * 1.02^{10}$$

Example 4

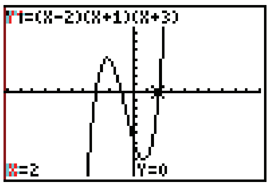
Generate the data set above using a spreadsheet tool. Use the fill command to produce cell formulas rapidly. Explain how the cell formulas in the spreadsheet are similar to the recursive rules given in Example 1.

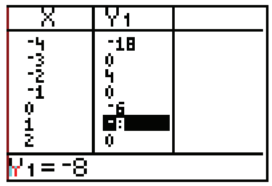
HSCE: A2.1.6

Identify the zeros of a function and the intervals where the values of a function are positive or negative, and describe the behavior of a function, as x approaches positive or negative infinity, given the symbolic and graphical representations.

Example 1

Find the zeroes of $y = (x - 2)(x + 1)(x + 3)$. Name the intervals of the function where the values of the function are positive or negative. Then graph the function.



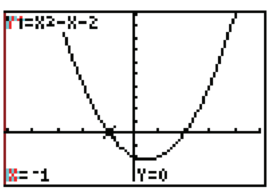


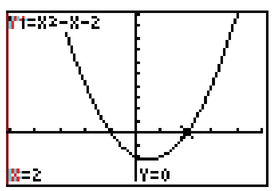
The zeros of the function are 2, -1, and -3.

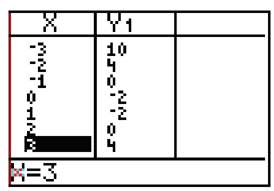
$$f(x)\ is \begin{cases} negative\ if\ x < -3 \\ positive\ if\ -3 < x < -1 \\ negative\ if\ -2 < x < 2 \\ positive\ if\ x > 2 \end{cases}$$

Example 2

Find the zeroes of $y = x^2 - x - 2$. Then write in factored form.







The zeros are x = -1, 2.

The factored form is $f(x) = (x+1)(x-2)$

HSCE: A2.1.7
Identify and interpret the key features of a function from its graph or its formula(e).

Example 1

$f(x) = 3x^2 + 2x + 1$
Identify the axis of symmetry, vertex (minimum), and the average rate of change from x=1 to x=3.

Enrichment
Find the derivative of the function.

Example 2
For the function $f(x) = \frac{5}{x}$, find the asymptotes and the type of symmetry from the graph. What happens to y as x approaches zero from the left and from the right? What happens when x approaches positive or negative infinity?

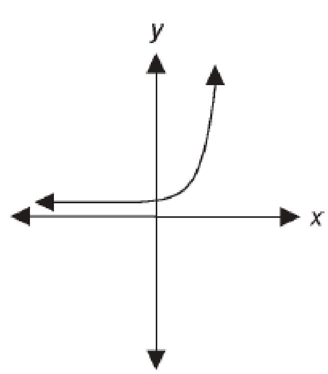
Example 3

Find the slope and y-intercept in the function $f(x) = 4x - 16$.

Example 4
For the function $f(x) = 2^x$, what is the asymptote?

Example 5
Compare and contrast the functions $f(x) = x^7$ and $f(x) = x^8$.

Example 6
Which of the following equations does this graph represent?



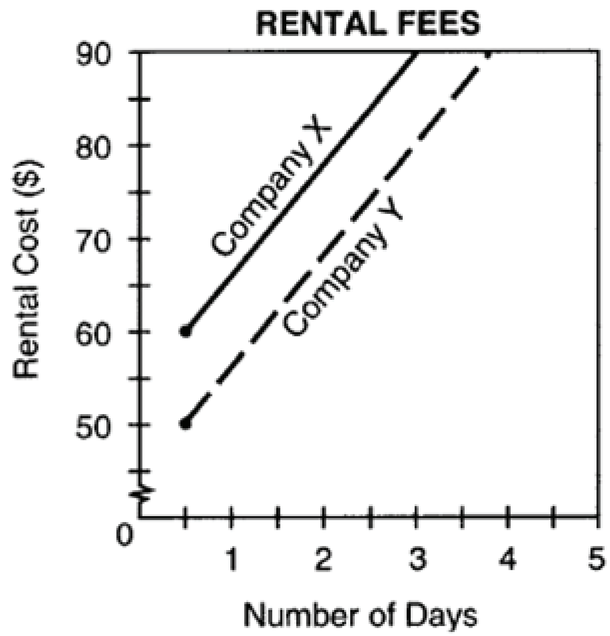
A $y = 3^x + 4$ **B** $y = 3x^3 + 4$

C $y = 3x + 4$ **D** $y = \frac{3}{x} + 4$

Source: MEAP Released Items, High School Test, 2002

Example 7

Marcia compared the cost of renting a word processor from two different companies. Both companies charged a deposit plus a daily fee. Marcia made a graph to compare the rental fees.



Based on the information in the graph, which statement is **true**?

A. Company X charges a higher daily fee.

B. Company Y charges a higher deposit.

C. Both companies charge the same daily fee.

D. Both companies charge the same deposit.

Source: MEAP Released Items, High School Test, 2001

Example 8

The equation for the curve shown in Figure 1 is $y=x^2$. Which of the following equations BEST represents the curve shown in Figure 2?

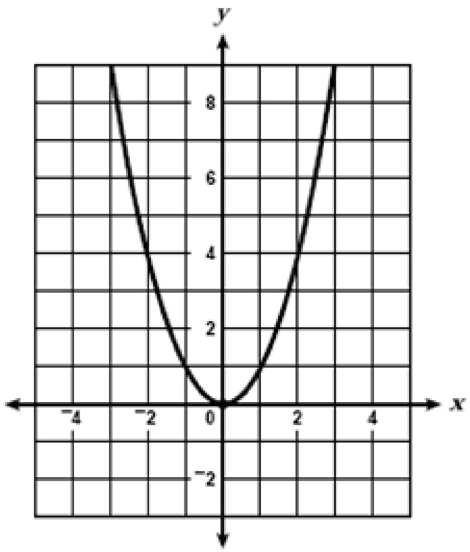


Figure 1

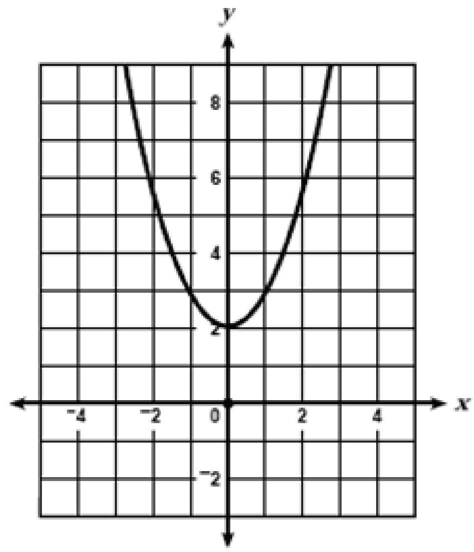


Figure 2

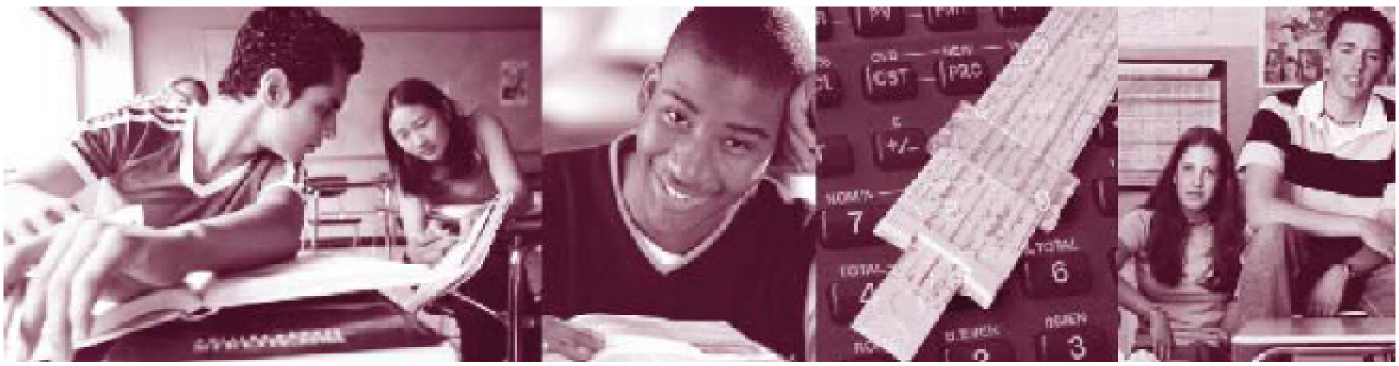
A. $y=\dfrac{x^2}{2}$ B. $y=2x^2$ C. $y=x^2+2$ D. $y=x^2-2$

Source: MEAP Released Items, High School Test, 2001

Michigan Department of Education
Betty Underwood, Interim Director
Office of School Improvement
www.michigan.gov/mde

High School Content Expectations

Clarification Document



MATHEMATICS

Algebra
Topic A2.2: Operations and Transformations with Functions

RIGOR • RELEVANCE • RELATIONSHIPS

Mathematics Clarification Document
Introduction

The Michigan Mathematics High School Content Expectations (HSCE) establish what every student is expected to know and be able to do by the end of high school and outline the parameters for receiving high school credit as recently mandated by the Michigan Merit Curriculum (MMC) legislation in the state of Michigan. In an effort to support mathematics teachers as they implement the Mathematic HSCE for all students, the Michigan Mathematics Leadership Academy (MMLA), a collaboration of the Michigan Mathematics and Science Center Network, the Michigan Council of Teachers of Education (MCTM) and the Michigan Department of Education, has developed this series of companion documents, with the goal of developing a common understanding of what the expectations mean. It is intended as a tool for mathematics educators and assessment developers to provide a clearer understanding of the breadth and depth of the HSCE.

Each document in the series represents one topic of the Mathematics HSCE and follows the same format:

- Information on the strand and standard where the topic is found. This information comes directly out of the HSCE document itself but may also include some further clarifying information.
- A list of the expectations in the topic including clarifying statements where needed.
- Background Information, Tools and Representations – a reference area that includes important formulas, properties, techniques and pedagogical tips for the topic in general.
- Assessable content intended as a technical guide for assessment developers, in addition to the information found in the rest of the document.
- A resource section that provides links to websites related to the topic.
- Clarifying Examples and Activities –include brief descriptions of classroom activities, assessment items and open-ended tasks all with the intent of providing further insights into the breadth and depth of each of the expectations. In many cases, suggested interventions for struggling students are provided.

Mathematics
Clarification for Topic A2.2: Operations and Transformations with Functions

Strand: A-Algebra

In the middle grades, students see the progressive generalization of arithmetic to algebra. They learn symbolic manipulation skills and use them to solve equations. They study simple forms of elementary polynomial functions such as linear, quadratic, and power functions as represented by tables, graphs, symbols, and verbal descriptions.

In high school, students continue to develop their “symbol sense” by examining expressions, equations, and functions, and applying algebraic properties to solve equations. They construct a conceptual framework for analyzing any function and, using this framework, they revisit the functions they have studied before in greater depth. By the end of high school, their catalog of functions will encompass linear, quadratic, polynomial, rational, power, exponential, logarithmic, and trigonometric functions. They will be able to reason about functions and their properties and solve multi-step problems that involve both functions and equation-solving. Students will use deductive reasoning to justify algebraic processes as they solve equations and inequalities, as well as when transforming expressions.

This rich learning experience in Algebra will provide opportunities for students to understand both its structure and its applicability to solving real-world problems. Students will view algebra as a tool for analyzing and describing mathematical relationships, and for modeling problems that come from the workplace, the sciences, technology, engineering, and mathematics.

STANDARD: A2 – FUNCTIONS

Students understand functions, their representations, and their attributes. They perform transformations, combine and compose functions, and find inverses. Students classify functions and know the characteristics of each family. They work with functions with real coefficients fluently.

Students construct or select a function to model a real-world situation in order to solve applies problems. They draw on their knowledge of families of functions to do so.

All of the topics in this standard are intended to frame the discussion about function families throughout both algebra courses.

Topic A2.2: Operations and Transformations with Functions

HSCE: A2.2.1 Combine functions by addition, subtraction, multiplication, and division.

Clarification: none

HSCE: A2.2.2 Apply given transformations to parent functions and represent symbolically.

Clarification: vertical or horizontal shifts, stretching or shrinking, or reflections about the *x*- and *y*-axes

HSCE: A2.2.3 Determine whether a function (given in tabular or graphical form) has an inverse and recognize simple inverse pairs.

Clarification: $f(x) = x^3$ and $g(x) = x^{1/3}$

***HSCE: A2.2.4** If a function has an inverse, find the expression(s) for the inverse.

***HSCE: A2.2.5** Write an expression for the composition of one function with another; recognize component functions when a function is a composition of other functions.

***HSCE: A2.2.6** Know and interpret the function notation for inverses and verify that two functions are inverses using composition.

Clarification: A2.2.4, A2.2.5 and A2.2.6 are recommended expectations. They represent content that is desirable and valuable for all students, but attention to these items should not displace or dilute the curricular emphasis of any of the required expectations. Teachers are encouraged to incorporate the recommended expectations into their instruction when their students have a solid foundation and are ready for enrichment or advanced learning.

Background Information, Tools, and Representations

- ❖ Graphing calculators should be used to allow students to see how changes to the parent functions results in transformations of the graph.
- ❖ Basically speaking, the process of finding an inverse is simply the swapping of the x and y coordinates.

Assessable Content

- ❖ Each of these expectations may be assessed in conjunction with a specific function family (see Standard A3 topics).
- ❖ Students should also demonstrate knowledge of these expectations independent of a specific function family.
- ❖ Assessments should be limited to appropriate course function families and at the appropriate level of difficulty. For instance, don't use a polynomial of degree 5 when a polynomial of degree 3 will work just as well.

Resources

Function Transformations / Translations
http://www.purplemath.com/modules/fcntrans.htm

Function Flyer
http://www.shodor.org/interactivate/activities/FunctionFlyer/?version=1.5.0_07&browser=Mozilla&vendor=Apple_Computer,_Inc.

Inverse Functions
http://regentsprep.org/Regents/mathb/7a3/inverselesson.htm

<u>**Clarifying Examples and Activities**</u>

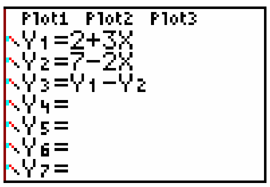
HSCE: A2.2.1
Combine functions by addition, subtraction, multiplication, and division.

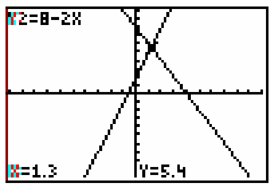
Example 1:

To solve a system of equations:

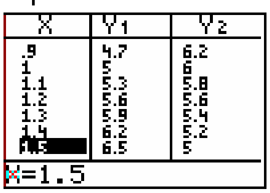
$y_1 = 2 + 3x$
$y_2 = 7 - 2x$

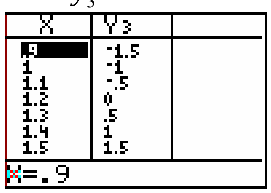
From the graph, students recognize where $y_1 < y_2$, $y_1 > y_2$ and $y_1 = y_2$. To find the solution for the system where $y_1 = y_2$, students may use the difference equation $y_3 = y_2 - y_1$ to explore the graphs as they approach the intersection point from the right and from the left.





Explain how the differences y_3 relate to the graphs of the two equations.



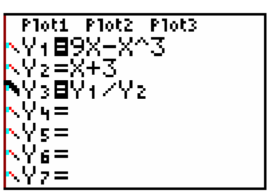


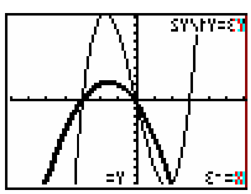
Example 2:

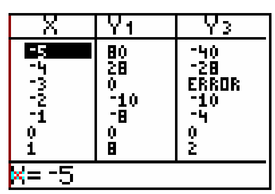
Combine functions defined in y_1, y_2 and y_3 into a single function
$y_4 = y_3 + y_2 + y_1$
(See A2.1.4 Piece-wise Functions)

Example 3:

To divide a polynomial by a binomial: $\dfrac{9x - x^3}{x+3}$







$y_3 = \dfrac{y_1}{y_2}$ defines the quotient.

Example 4:

- Fill in the blank in the sentences below with ALWAYS, SOMETIMES, or NEVER. Give examples to support your answer.
  The sum of two linear functions is ______ a linear function.
  The product of two linear functions is ______ a linear function.
  This activity gives students the opportunity to analyze the structure of operations on functions. (Note: This example also provides an opportunity to talk about when or whether examples are sufficient justification for mathematical statements.)

(From MiCLiMB)

HSCE: A2.2.2
Apply given transformations to parent functions and represent symbolically.

Example 1:
$y = a\left(\frac{x-b}{c}\right)^2 + d$ represents a transformation of the parent function $y = x^2$.

Which constants (a, b, c, or d) for the given function define the geometric transformations below?

Vertical stretch/compression ____________

Horizontal stretch/compression ____________

Horizontal translation ____________

Vertical translation ____________

HSCE: A2.2.3
Determine whether a function (given in tabular or graphical form) has an inverse and recognize simple inverse pairs.

Example 1:

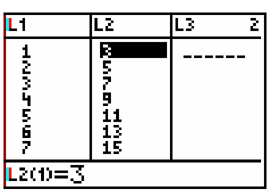
Determine the function $f(x)$, and then complete the following table.

x	$f(x)$	$f^{-1}(x)$
2	**6**	**2**
4	**12**	
8		**8**
		12
	30	

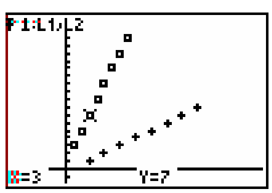
Example 2:

Enter linear data in List 1 and List 2. Find the line of best fit for (Line 1, Line 2). What is the meaning of the coefficients in the best fit line? Find the line of best fit for (Line 1, Line 2) where the output and input reversed. What is the meaning of the coefficients in the best fit line. Find the line of best fit for (Line 2, Line 1). What is the meaning of the coefficients in the this best fit line. How are the two linear equations related.

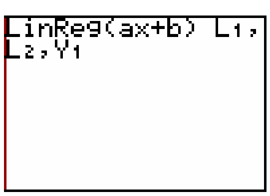
Linear Data

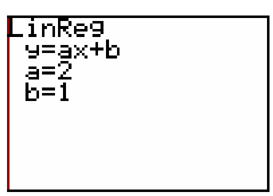


Graph of (L1,L2) and (L2,L1)



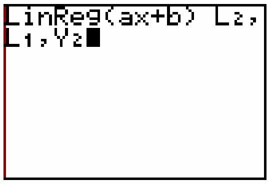
Regression Equation for (L1,L2)

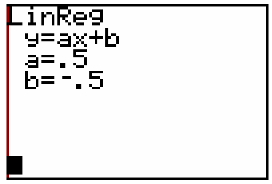




y=2x+1

Regression Equation for (L1,L2)





y=.5x-.5 (inverse equation)

Discuss the order of operations in y=2x+1. Compare to order of operations in y=.5x-.5

$y = 2x + 1$, therefore $x = \frac{y-1}{2} = \frac{y}{2} - \frac{1}{2}$

Suggestions for differentiation

Enhancement

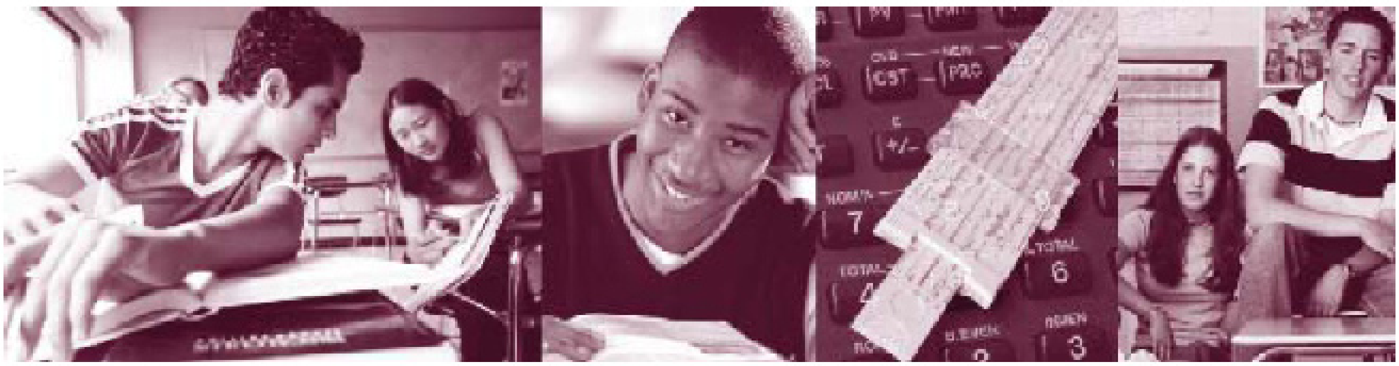
Recommended HSCE: A2.2.4 If a function has an inverse, find the expression(s) for the inverse.

Recommended HSCE: A2.2.6 Know and interpret the function notation for inverses and verify that two functions are inverses using composition.

Michigan Department of Education
Betty Underwood, Interim Director
Office of School Improvement
www.michigan.gov/mde

High School Content Expectations

Clarification Document



MATHEMATICS

Algebra
Topic A2.3: Representation of Functions

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Michigan State Board of Education

Mathematics Clarification Document
Introduction

The Michigan Mathematics High School Content Expectations (HSCE) establish what every student is expected to know and be able to do by the end of high school and outline the parameters for receiving high school credit as recently mandated by the Michigan Merit Curriculum (MMC) legislation in the state of Michigan. In an effort to support mathematics teachers as they implement the Mathematic HSCE for all students, the Michigan Mathematics Leadership Academy (MMLA), a collaboration of the Michigan Mathematics and Science Center Network, the Michigan Council of Teachers of Education (MCTM) and the Michigan Department of Education, has developed this series of companion documents, with the goal of developing a common understanding of what the expectations mean. It is intended as a tool for mathematics educators and assessment developers to provide a clearer understanding of the breadth and depth of the HSCE.

Each document in the series represents one topic of the Mathematics HSCE and follows the same format:

- Information on the strand and standard where the topic is found. This information comes directly out of the HSCE document itself but may also include some further clarifying information.
- A list of the expectations in the topic including clarifying statements where needed.
- Background Information, Tools and Representations – a reference area that includes important formulas, properties, techniques and pedagogical tips for the topic in general.
- Assessable content intended as a technical guide for assessment developers, in addition to the information found in the rest of the document.
- A resource section that provides links to websites related to the topic.
- Clarifying Examples and Activities –include brief descriptions of classroom activities, assessment items and open-ended tasks all with the intent of providing further insights into the breadth and depth of each of the expectations. In many cases, suggested interventions for struggling students are provided.

Mathematics
Clarification for Topic A2.3: Representations of Functions

Strand: A-Algebra

In the middle grades, students see the progressive generalization of arithmetic to algebra. They learn symbolic manipulation skills and use them to solve equations. They study simple forms of elementary polynomial functions such as linear, quadratic, and power functions as represented by tables, graphs, symbols, and verbal descriptions.

In high school, students continue to develop their "symbol sense" by examining expressions, equations, and functions, and applying algebraic properties to solve equations. They construct a conceptual framework for analyzing any function and, using this framework, they revisit the functions they have studied before in greater depth. By the end of high school, their catalog of functions will encompass linear, quadratic, polynomial, rational, power, exponential, logarithmic, and trigonometric functions. They will be able to reason about functions and their properties and solve multi-step problems that involve both functions and equation-solving. Students will use deductive reasoning to justify algebraic processes as they solve equations and inequalities, as well as when transforming expressions.

This rich learning experience in Algebra will provide opportunities for students to understand both its structure and its applicability to solving real-world problems. Students will view algebra as a tool for analyzing and describing mathematical relationships, and for modeling problems that come from the workplace, the sciences, technology, engineering, and mathematics.

STANDARD: A2 – FUNCTIONS

Students understand functions, their representations, and their attributes. They perform transformations, combine and compose functions, and find inverses. Students classify functions and know the characteristics of each family. They work with functions with real coefficients fluently.

Students construct or select a function to model a real-world situation in order to solve applies problems. They draw on their knowledge of families of functions to do so.

All of the topics in this standard are intended to frame the discussion about function families throughout both algebra courses.

Topic A2.3: Representations of Functions

A.2.3 discusses distinguishing one family from another. A2.4-A2.10 can be thought of as subheadings to this topic. They apply **A2.3** to each family and address any characteristics that are specific to each type of function.

HSCE: A2.3.1 Identify a function as a member of a family of functions based on its symbolic or graphical representation. Recognize that different families of functions have different asymptotic behavior.

Clarification:

- Students should look at an algebraic expression or a graph and recognize it as a variation of a parent function.
- The asymptotic behavior only applies to functions such as exponential, rational, and logarithmic

HSCE: A2.3.2 Describe the tabular pattern associated with functions having constant rate of change (linear); or variable rates of change.

Clarification: This is only to distinguish between linear and nonlinear functions.

HSCE: A2.3.3 Write the general symbolic forms that characterize each family of functions.

Clarification: none

Background Information, Tools, and Representations

- General symbolic forms:
  Linear: $f(x) = mx + b$; *a+bx or ax+b* (Ti-83 notation)
  Quadratic: $g(x) = ax^2 + bx + c$
  Exponential: $h(x) = a(b)^x$
  Power: $j(x) = kx^p$ where *k* and *p* are constants
  Polynomial: $p(x) = a_n x^n + a_{n-1} x^{n-1} + ... + a_1 x + a_0$
- Technology should be used to allow students to quickly and easily see the differences in graphs, tables and symbolic forms between function families.
- A graphing program or calculator can be used to identify changes in graphs given different coefficients, sums and differences on the parent function.
- Comparison of Linear and Exponential Functions (see also the comparison table on page 8)
  - In a linear equation bx represents additive change; (a+b+b+b+b+……….)
    - the slope **b** is the constant rate of change between any two points $(x_1, y_2) and (x_2, y_2) = \dfrac{(y_2 - y_1)}{(x_2 - x_1)}$
  - In an exponential equation, ab^x, b^x represents multiplicative change: (a*b*b*b*……….)

    Growth **factor b** = 100% + growth **rate** (percent)
    Example: Value of savings account is growing by 2.5% per year;
    the growth factor is 1.025 and $y = balance(1.025)^x$.

    Decay **factor b** = 100%-decay **rate** (percent)
    Example: Value of the dollar is declining each year by 3%;
    the decay factor is 97% and $y = 1.00(.97)^x$.

  - Describe the rate of change

    The **average** rate of change between data points of an exponential function will not be constant.

    The **average** rate of change between any two non-linear points is the slope of the secant line that contains the two points.

    $$\frac{(y_2 - y_1)}{(x_2 - x_1)}$$

    As students conceptually explore the similarities and differences between linear and exponential functions, be sure they use correct terminology for constant rate of change (slope) and average rate of change (slope of secant line).
- Recognize patterns by matching given tables and graphs

Assessable Content

- **A2.3.1** and **A2.3.3** may be assessed in conjunction with a specific function family (see Standard A3 topics).
- Students should also demonstrate knowledge of these expectations independent of a specific function family.
- Assessments should be limited to appropriate course function families and at the appropriate level of difficulty. For instance, don't use a polynomial of degree 5 when a polynomial of degree 3 will work just as well.

Resources

Function Flyer
http://www.shodor.org/interactivate/activities/FunctionFlyer/?version=1.6.0_01&browser=MSIE&vendor=Sun_Microsystems_Inc.

Determining the average slope of a hill using a topographic map is fairly simple. Slope can be given in two different ways, a percent gradient and an angle of the slope. The initial steps to calculating slope either way are the same.
http://geology.isu.edu/geostac/Field_Exercise/topomaps/slope_calc.htm

Context: Population Growth Websites
Population grows in the same way that money grows when it's left to compound interest in a bank. With money, growth comes through accumulating interest upon interest. The interest payments you accumulate eventually earn interest, increasing your money. With population growth, new members of the population eventually produce other new members of the population. The population increases exponentially as time passes.
http://www.learner.org/exhibits/dailymath/population.html

This applet is designed to show how the population growth drives the demand for power, which in turn requires more power generation capacity, which in turn has an environmental consequence.
http://jersey.uoregon.edu/vlab/ExponentialGrowth/

Population Connection's 7-minute video, World Population is a graphic simulation of human population growth. As the years roll by on a digital clock from 1 A.D. to 2030, dots light up on an illustrated map to represent millions of people added to the population. Historic references on the screen place population changes in context.
http://www.populationeducation.org/index.php?option=com_content&task=view&id=175&Itemid=10

The "Rule of 70" for Exponential Growth
Predict the doubling time of an investment. *70/n years; n =% growth rate*
http://mmcconeghy.com/students/supscruleof70.html

Clarifying Examples and Activities

HSCE: A2.3.1

Identify a function as a member of a family of functions based on its symbolic or graphical representation. Recognize that different families of functions have different asymptotic behavior

Example 1:

Group project
Each group creates a poster that shows the context, equation, table, and graph of a given function.

- Compare all of the posters and put the functions into groups having common features.
- Each function group summarized the common patterns of their family of functions.
- Summarize and share your descriptions with the whole class.
- Individually, each student records the summaries.

NOTE: The goal is to give students an opportunity to analyze characteristics of functions, not necessarily for them to generate the exact classes.

Example 2:

Identify an appropriate function family to model a set of bivariate data.

- The height of a kicked football as a function of elapsed time (quadratic).
- Number of bacteria growing as a function of elapsed time (exponential increase).
- Cost of parking a car at the airport during a trip (linear increasing or step function).
- The price of gas if it grows by $0.16 per week (linear increase)
- The speed of personal computers if the speed doubles every three years.(exponential increase)
- The area of a rectangle that is 6 inches narrower than it is long. (quadratic)
- The value of an investment that drops by one-third n times in a row (exponential decrease)
- The rate of absorption of one dose of insulin in the human body. (exponential decrease)
- A function that has x-intercepts at -3, 0, and 4, and -5 and no asymptotes (quartic polynomial)

Example 3:

Identify the parent function of $g(x) = \dfrac{5}{x+1}$ and describe the characteristics of the graph of g(x)

HSCE: A2.3.2

Describe the tabular pattern associated with functions having constant rate of change (linear); or variable rates of change.

Example 1:

Which of the following tables describe a linear relationship? Explain.

X	Y1	Y2
-3	9	-1.5
-2	7	-1
-1	5	-.5
0	3	0
1	1	.5
2	-1	1
3	-3	1.5

X= -3

X	Y3	Y4
-3	9	6
-2	4	5.6667
-1	1	5.3333
0	0	5
1	1	4.6667
2	4	4.3333
3	9	4

Y4=6

Suggestions for differentiation

Intervention:

Given a linear table, a quadratic table, and an exponential table, students will describe how the ***y*** changes in each table and write the symbolic form of the matching function.

HSCE: A2.3.3

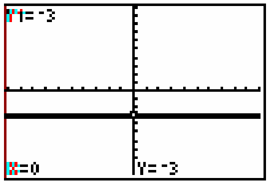
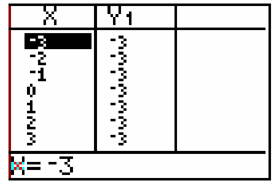
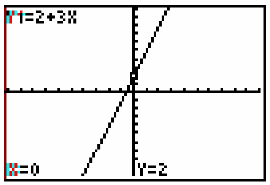
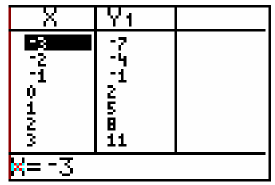
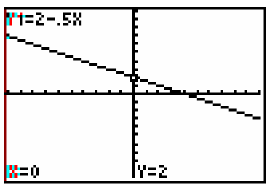
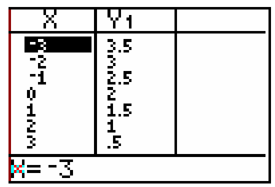
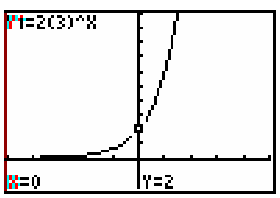
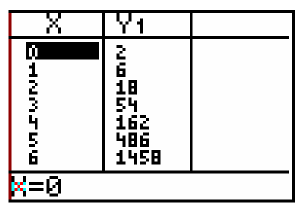
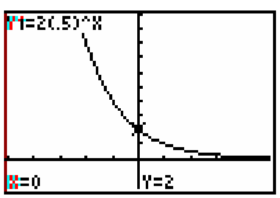
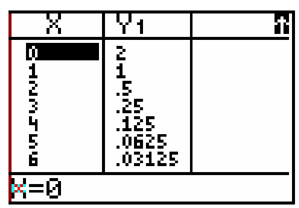
Write the general symbolic forms that characterize each family of functions.

Example 1:

Function Sort

1. Place four function family names at eye level across the wall; linear, quadratic, exponential, polynomial and power.
2. Explain to students that they will apply the multiple representations of function families in this activity.
3. Give each student a paper containing a graph or table or equation.
4. Each student will need to find the rest of their team (3) to complete a function family.
5. Students place their graph, table and equation with the appropriate function family name.
6. Once all function papers are sorted on the wall, have each student in each group summarize the patterns they used to find their function family.
7. As a whole group, compare and contrast the graphs, tables and equations for each function family.
   - Are there any similarities between the quadratic graphs and the power graphs?
   - How can the tables help you identify the type of function?
   - How are the patterns in the graphs reflected in the tables?
8. Students record summaries in their journals

Exponential and Linear Functions

	Graph	Rate of Change in y values	Table	X intercept	Y intercept	Domain X values	Range Y values
Linear y=a+bx slope of b=0 constant function y=-3		Slope of zero Next = now+0 starting at -3 *(see A2.1 for explanation of now-next recursive rule)*	 X: -3, -2, -1, 0, 1, 2, 3 Y1: -3, -3, -3, -3, -3, -3, -3 X=-3	none	(0,a)	All real numbers	y=a
Linear y=a+bx slope of b>0 linear increasing y=2+3x		Increasing at a constant rate of 3 units Next = now+3 starting at 2	 X: -3, -2, -1, 0, 1, 2, 3 Y1: -7, -4, -1, 2, 5, 8, 11 X=-3	$\left(-\frac{a}{b},0\right)$	(0,a)	All real numbers	All real numbers
Linear y=a+bx slope of b<0 linear decreasing y=2-.5x		Decreasing at a constant rate of -.5 units Next = now + -.5 starting at 2	 X: -3, -2, -1, 0, 1, 2, 3 Y1: 3.5, 3, 2.5, 2, 1.5, 1, .5 X=-3	$\left(-\frac{a}{b},0\right)$	(0,a)	All real numbers	All real Numbers
Exponential Growth $y = ab^x$ $b > 1$ *Growth factor b $y = 2(3)^x$		Increasing at an increasing rate of 300% Next=now*3 starting at 2	 X: 0, 1, 2, 3, 4, 5, 6 Y1: 2, 6, 18, 54, 162, 486, 1458 X=0	None x-axis is asymptote	(0,a)	all real numbers	y>0
Exponential Decay $y = ab^x$ $0 < b < 1$ *Decay Factor b $y = 2(.5)^x$		Decreasing at a decreasing rate of 50% Next = now*.5 starting at 2	 X: 0, 1, 2, 3, 4, 5, 6 Y1: 2, 1, .5, .25, .125, .0625, .03125 X=0	None x-axis is asymptote	(0,a)	all real numbers	y>0

Michigan Department of Education
Betty Underwood, Interim Director
Office of School Improvement
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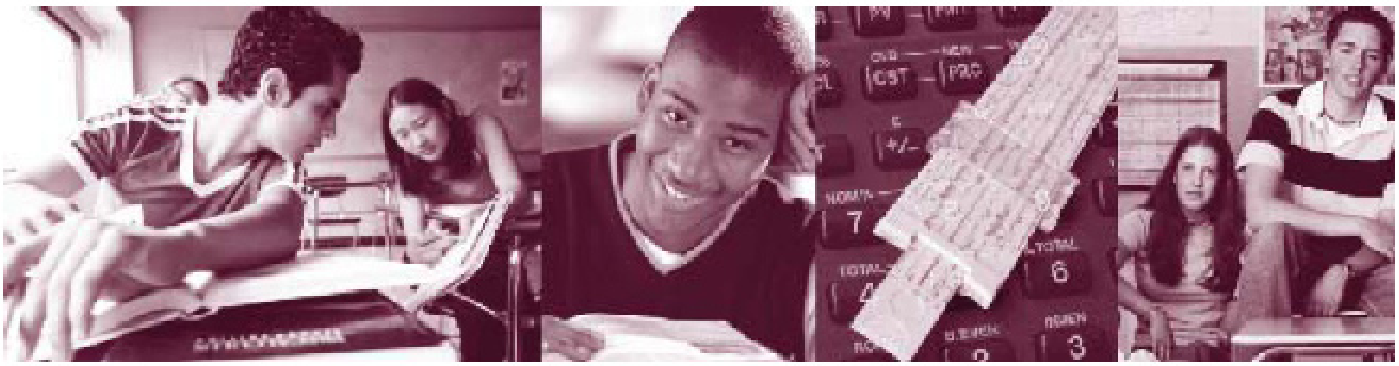
High School Content Expectations

Clarification Document



MATHEMATICS

Algebra

Topic A2.4: Models of Real-World Situations Using Families of Functions

Michigan State Board of Education

Mathematics Clarification Document
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Mathematics
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Strand: A-Algebra

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In high school, students continue to develop their “symbol sense” by examining expressions, equations, and functions, and applying algebraic properties to solve equations. They construct a conceptual framework for analyzing any function and, using this framework, they revisit the functions they have studied before in greater depth. By the end of high school, their catalog of functions will encompass linear, quadratic, polynomial, rational, power, exponential, logarithmic, and trigonometric functions. They will be able to reason about functions and their properties and solve multi-step problems that involve both functions and equation-solving. Students will use deductive reasoning to justify algebraic processes as they solve equations and inequalities, as well as when transforming expressions.

This rich learning experience in Algebra will provide opportunities for students to understand both its structure and its applicability to solving real-world problems. Students will view algebra as a tool for analyzing and describing mathematical relationships, and for modeling problems that come from the workplace, the sciences, technology, engineering, and mathematics.

STANDARD: A2 – FUNCTIONS

Students understand functions, their representations, and their attributes. They perform transformations, combine and compose functions, and find inverses. Students classify functions and know the characteristics of each family. They work with functions with real coefficients fluently.

Students construct or select a function to model a real-world situation in order to solve applies problems. They draw on their knowledge of families of functions to do so.

All of the topics in this standard are intended to frame the discussion about function families throughout both algebra courses.

Topic A2.4: Models of Real-world Situations Using Families of Functions

Clarification: Students construct or select a function to model a real-world situation in order to solve applied problems. They draw on their knowledge of classes of functions to do so. They solve applied problems involving linear programming.

A2.4.1, A2.4.2, and A2.4.3 should not be treated as separate; they describe the sequence of steps used in solving any modeling problem. This standard requires translating verbal statements into symbolic forms, solving the problem symbolically, and then interpreting the solution verbally. Real-world modeling problems should be incorporated into the curriculum as part of the study of each family of functions. After having studied several families, a student should be able to read a problem presented verbally and know from the context which function family to choose, how to represent that family symbolically, and how to manipulate the symbols to get the answer.

HSCE: A2.4.1 Identify the family of function best suited for modeling a given real-world situation.

Clarification: e.g., quadratic functions for motion of an object under the force of gravity; exponential functions for compound interest; trigonometric functions for periodic phenomena. Given the example "*An initial population of 300 people grows at 2% per year. What will the population be in 10 years?*" students should recognize that the appropriate general function is exponential ($P = P_0a^t$)].

HSCE: A2.4.2 Adapt the general symbolic form of a function to one that fits the specifications of a given situation by using the information to replace arbitrary constants with numbers.

Clarification: In the example above, substitute the given values P_0 = 300 and a = 1.02 to obtain $P = 300(1.02)^t$.

HSCE: A2.4.3 Using the adapted general symbolic form, draw reasonable conclusions about the situation being modeled.

Clarification: In the example above, the exact solution is 365.698, but for this problem, an appropriate approximation is 365.

***HSCE: A2.4.4** Use methods of linear programming to represent and solve simple real-life problems.

Clarification: This is a recommended expectation. It represents content that is desirable and valuable for all students, but attention to these items should not displace or dilute the curricular emphasis of any of the required expectations. Teachers are encouraged to incorporate the recommended expectations into their instruction when their students have a solid foundation and are ready for enrichment or advanced learning.

Background Information, Tools, and Representations

Assessable Content

- These expectations should not be tested separately from each other (see topic clarification above). A scenario should be set up where students are asked to read a problem and then choose the appropriate function family, represent that family symbolically, and manipulate the symbols to get the answer.
- This topic should be assessed in conjunction with a specific function family (see Standard A3 topics).
- Students should also demonstrate knowledge of these expectations independent of a specific function family.
- Assessments should be limited to appropriate course function families and at the appropriate level of difficulty. For instance, don't use a polynomial of degree 5 when a polynomial of degree 3 will work just as well.

Resources

Clarifying Examples and Activities

Example 1

Linear Example:
A cellular phone company has a plan with a basic monthly cost of $59.98 with an allowance of 600 minutes. Minutes beyond the allowance are billed at a rate of $0.10 per minute. (Ignore free night and weekend minutes.)

- Write a rule to calculate the total monthly charge if a person talks for x minutes. Assume x>600.
- Calculate the bill for a person who talks 710 minutes in a month.
- Determine the number of minutes a person would have talked if their bill were $66.98.

Extension: Compare two plans with different parameters

- Choose values that do not occur at integer values of x
- Create a piece-wise rule for all x (before and after 600 min)

Example 2

Exponential Example:
A city has a population that declines about 4% per year. If the city has a population of 152,000 in 2002:

- Write a rule to estimate the population x years after 2002.
- Calculate an estimate for the population in 2010.
- Determine the approximate year that the population 85,830.

Extension: Ask for values before 2002

- Give an example of a smaller city whose population is growing.
- Ask when smaller city overtakes larger city.
- Compare to a city with a constant (linear) growth/decay rate and use technology to estimate population intersection.

Example 3

Quadratic Example:
The stopping distance increases in proportion with the square of velocity. A car on a test track is accelerated to a velocity of 50 mph and then brought to a stop without skidding. The stopping distance is 125 ft.

- Write a rule to calculate the stopping distance for a velocity of x mph.
- Calculate the stopping distance for a car traveling 65 mph.
- Determine how fast a car is going if it’s stopping distance is 200 ft.

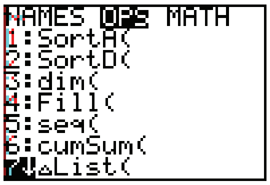
Extension:

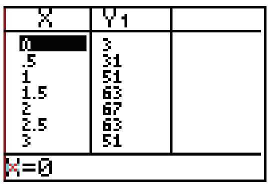
- See www.arachnold.com/lutusp/auto.html for factoring in reaction time to link to A2.2.1 addition of functions.

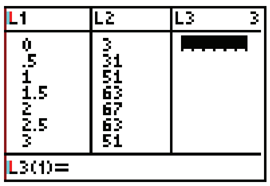
Example 4

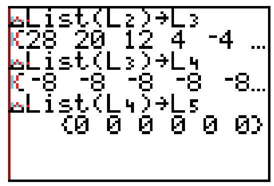
If you throw a ball straight up in the air, the equation $h = -16t^2 + 64t + 3$ models the height (feet) of the ball at any time t (seconds). Create a table of (t,h) values with t values from 0 to 4 seconds in increments of .5 seconds.

Explore the rate of change patterns of 2nd degree power or quadratic equations. Use the Δ *List* function of the graphing calculator to find the first and second differences of the y values. *It can be discovered that for an n-degree polynomial, the nth difference is constant and the (n+1)th difference is zero.*









Discuss the meaning of the first difference (L3). $\dfrac{\Delta height}{.5\sec}$ = velocity (linear)

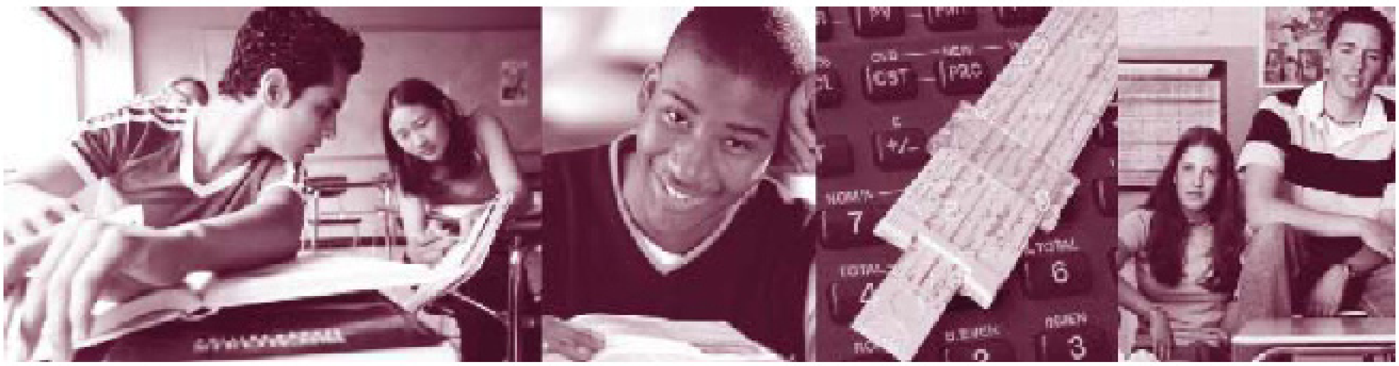
Discuss the meaning of the second difference(L4). $\dfrac{\Delta velocity}{.5\sec}$ = acceleration (constant)

If the 2nd differences are the same, then the relationship must be quadratic or power. This is the precursor to studying derivative functions in calculus.

Michigan Department of Education
Betty Underwood, Interim Director
Office of School Improvement
www.michigan.gov/mde

High School Content Expectations

Clarification Document



MATHEMATICS

Algebra
Topic A3.1: Lines and Linear Functions

Michigan State Board of Education

Mathematics Clarification Document
Introduction

The Michigan Mathematics High School Content Expectations (HSCE) establish what every student is expected to know and be able to do by the end of high school and outline the parameters for receiving high school credit as recently mandated by the Michigan Merit Curriculum (MMC) legislation in the state of Michigan. In an effort to support mathematics teachers as they implement the Mathematic HSCE for all students, the Michigan Mathematics Leadership Academy (MMLA), a collaboration of the Michigan Mathematics and Science Center Network, the Michigan Council of Teachers of Education (MCTM) and the Michigan Department of Education, has developed this series of companion documents, with the goal of developing a common understanding of what the expectations mean. It is intended as a tool for mathematics educators and assessment developers to provide a clearer understanding of the breadth and depth of the HSCE.

Each document in the series represents one topic of the Mathematics HSCE and follows the same format:

- Information on the strand and standard where the topic is found. This information comes directly out of the HSCE document itself but may also include some further clarifying information.
- A list of the expectations in the topic including clarifying statements where needed.
- Background Information, Tools and Representations – a reference area that includes important formulas, properties, techniques and pedagogical tips for the topic in general.
- Assessable content intended as a technical guide for assessment developers, in addition to the information found in the rest of the document.
- A resource section that provides links to websites related to the topic.
- Clarifying Examples and Activities –include brief descriptions of classroom activities, assessment items and open-ended tasks all with the intent of providing further insights into the breadth and depth of each of the expectations. In many cases, suggested interventions for struggling students are provided.

Mathematics
Clarification for Topic A3.1: Lines and Linear Functions

Strand: A-Algebra

In the middle grades, students see the progressive generalization of arithmetic to algebra. They learn symbolic manipulation skills and use them to solve equations. They study simple forms of elementary polynomial functions such as linear, quadratic, and power functions as represented by tables, graphs, symbols, and verbal descriptions.

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This rich learning experience in Algebra will provide opportunities for students to understand both its structure and its applicability to solving real-world problems. Students will view algebra as a tool for analyzing and describing mathematical relationships, and for modeling problems that come from the workplace, the sciences, technology, engineering, and mathematics.

STANDARD: A3 – FAMILIES OF FUNCTIONS

Students study the symbolic and graphical forms of each function family. By recognizing the unique characteristics of each family, students can use them as tools for solving problems or for modeling real-world situations.

Topic A3.1 Lines and Linear Functions

HSCE: A3.1.1 Write the symbolic forms of linear functions (standard, point-slope, and slope-intercept) given appropriate information, and convert between forms.

Clarification: The emphasis of this expectation is to be able to convert from standard form to slope-intercept form.

HSCE: A3.1.2 Graph lines (including those of the form x = h and y = k) given appropriate information.

Clarification: Appropriate information means e.g. slope and intercept, slope and a point on the line, two points on the line, etc. Students should exhibit knowledge of equations of the form $x=h$ and $y=k$ and their vertical and horizontal line graphs.

HSCE: A3.1.3 Relate the coefficients in a linear function to the slope and *x*- and *y*-intercepts of its graph.

Clarification: Students should be able to solve for the *x* and *y* intercepts of any linear function. They should recognize the effect of the parameters *m* and *b* in the slope-intercept form. Given a linear function in standard form students should convert it to slope-intercept form to understand the relationship of the coefficients.

HSCE: A3.1.4 Find an equation of the line parallel or perpendicular to given line, through a given point; understand and use the facts that non-vertical parallel lines have equal slopes, and that non-vertical perpendicular lines have slopes that multiply to give -1.

Clarification: The idea of perpendicular lines is something that students should be aware of but it is not the critical part of this expectation. The important part is that of parallel lines have equal slopes.

Background Information, Tools, and Representations

- Symbolic forms of linear functions:
  - Standard: $Ax + By = C$, where $B \neq 0$
  - point-slope: $y_2 - y_1 = m(x_2 - x_1)$
  - slope intercept: $y = mx + c$

Assessable Content

- Incorporate concepts from A1 and A2 into assessments of this topic whenever possible.
- Assessments of this topic should include at least one situation where students are required to use linear functions to model real-world situations.
- On the ACT, never assume that two lines are parallel or perpendicular just because they look that way in a diagram. If the lines are parallel or perpendicular, the ACT will tell you so. (Perpendicular lines can be indicated by a little square located at the place of intersection, as in the diagram in A3.1.4.)
  http://www.sparknotes.com/testprep/books/act/chapter10section5.rhtml

Resources

For review of the forms of linear equation, go to:
http://www.themathpage.com/alg/slope-of-a-line-2.htm - point

Pedal Power
In this lesson, students investigate slope as a rate of change. Students compare, contrast, and make conjectures based on distance-time graphs for three bicyclists climbing to the top of a mountain.
http://illuminations.nctm.org/LessonDetail.aspx?id=L586

Movie Lines
This lesson allows students to apply their knowledge of linear equations and graphs in an authentic situation. They plot data points corresponding to the cost of DVD rentals and interpret the results.
http://illuminations.nctm.org/LessonDetail.aspx?id=L629

Exploring Linear Data Lab
Students model linear data in a variety of settings that range from car repair costs to sports to medicine. Students work to construct scatterplots, interpret data points and trends, and investigate the notion of line of best fit. *X*- and *y*- intercepts are interpreted in context.
http://illuminations.nctm.org/LessonDetail.aspx?id=L298

Cabri™ Jr. Activity 23: Investigating the Slopes of Parallel and Perpendicular Lines
http://education.ti.com/educationportal/activityexchange/Activity.do?cid=US&aId=6880

Use graph paper folding activities to create parallel and perpendicular lines. Using the coordinates of points on the folding lines, students discover the patterns of the slopes of parallel and perpendicular lines.
www.vidyaonline.net/arvindgupta/paperfolding.pdf

Clarifying Examples and Activities

HSCE: A3.1.1
Write the symbolic forms of linear functions (standard, point-slope, and slope-intercept) given appropriate information and convert between forms.

Example 1
Given the standard form of an equation $2x+3y=6$, which of the following is the slope-intercept form of the equation?

a. $x=-\frac{3}{2}y+3$
b. $3y=6-2x$
c. $2x=-3y+6$
d. $y=-\frac{2}{3}x+6$
e. $y=-\frac{2}{3}x+2$

Example 2
The following represent responses of six students to a given task in Algebra I. Which students have equivalent equations for the solution? Provide your reasoning.

Ruth: $3x + 6y = 15$
Anne: $y = -2x+15$
Max: $6x + 3y = 15$
Ahmad: $x + 2y = 5$
Taniqua: $y = -2x + 5$;
Debbie: $y - 5 = -2(x - 5)$

Example 3
Change from standard form to slope-intercept form. Identify the slope and *y*- intercept.

$$ax+by=c$$
$$by=c-ax$$
$$y=\frac{c-ax}{b}$$
$$y=\frac{c}{b}-\frac{ax}{b}$$
$$y=\frac{c}{b}-\frac{a}{b}x$$

$(0,c/b)$ is the *y*-intercept. The slope is $-a/b$.

HSCE: A3.1.2
Graph lines (including those of the form $x = h$ and $y = k$) given appropriate information.

Example 1
Graph each of the following linear situations.
Identify the slope, the y-intercept, or point(s) on graph that are given.
Find the equation of each graph.

- A car enters the expressway at 4:00 p.m. and reaches a cruising speed of 68 miles per hour at 4:05 p.m. The car remains at this speed for the next 3 hours.
    - Graph this situation on a time versus speed graph from 4:05 p.m, until 7:05 p.m..
    - Graph this situation on a time versus distance travelled graph from 4:05 p.m. until 7:05 p.m.
- At mile marker 102, a car on M-69 is traveling at a constant rate of speed. It takes the car 18 minutes to reach mile marker 123.
- At a reading of 74,256 miles on the odometer, Mike fills the gas tank of his automobile. At 74,396 miles, Mike stops again for fuel. It takes 7 gallons of gasoline to fill the tank.

Suggestions for differentiation

Intervention

- Use the graphing calculator technology and a CBL or CBR motion detector. Using the CBR/Ranger Applications programs of the TI-83/84, students walk to match a DIST MATCH graph. Linear piecewise graphs are generated in this program. Students find the velocity (slope) of each segment of the walk and use it to describe the walk.
- Use the Pedal Power or Movie Line activity (see Resources)

HSCE: A3.1.3
Relate the coefficients in a linear function to the slope and *x*- and *y*-intercepts of its graph.

Example 1
You have been hired by The Chair Company that makes different types of stacking chairs. For each type of stacking chair that the company makes, the company needs to know the heights of different stacks of chairs. Your task is to provide this information. Use a set of stacking chairs available in your school.

1. Record their measurements (number of chairs and the height of the stack) in a table.
2. Make a scatterplot of the data.
3. Find a line of "best fit" and graph it on the scatterplot and describe how you found this line.
4. How does the line of best fit describe the situation being modeled?
5. Predict how many stacked chairs would fit through a doorway that is 200 cm high and explain how you made your prediction.
6. Predict how tall a stack of 25 chairs would be and explain how you made your prediction.
7. Describe the rule you would use to find the height of n stacked chairs.

This activity could also be done with stacking cups (foam coffee cups)

HSCE: A3.1.4
Find an equation of the line parallel or perpendicular to given line through a given point. Understand and use the facts that non-vertical parallel lines have equal slopes and that non-vertical perpendicular lines have slopes that multiply to give -1.

Example 1
Graph the following lines on the same coordinate axes.
Label each line with its equation.

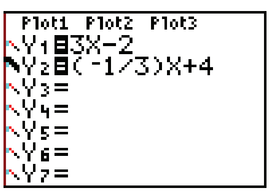
a. $y = 3x + 5$
b. $y = 2x + 5$
c. $y = 3x + 2$
d. $y = 5x + 3$
e. $y = -2x + 5$

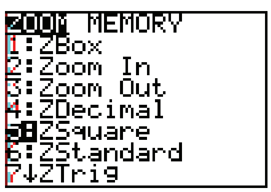
List the similarities and differences. As a class discuss your findings. What two lines appear to be parallel? What do you notice about their equations? Graph another line with that same characteristic. Does that confirm your conjecture?

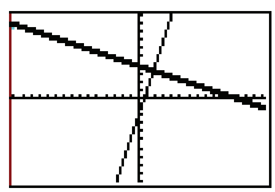
Example 2
Using the graphing calculator, students write four equations whose intersection points create the vertices of a rectangle, of a square.

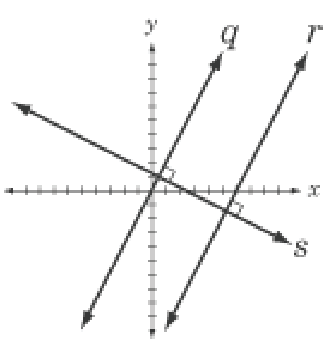
Using the graphing calculator, students write four equations whose intersection points create the vertices of a parallelogram.

Note: If using the graphing calculator, a square window will be required to make the lines appear perpendicular. Use Zoom 5.









Suggestions for differentiation

Intervention

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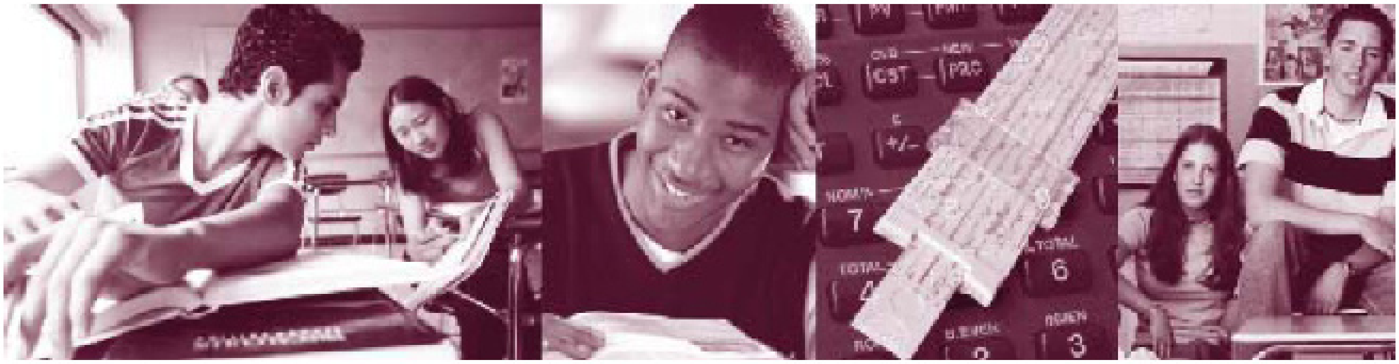
Use graph paper folding activities to create parallel and perpendicular lines (see Resources).

Michigan Department of Education
Betty Underwood, Interim Director
Office of School Improvement
www.michigan.gov/mde

High School Content Expectations

Clarification Document



MATHEMATICS

Algebra

Topic A3.2: Exponential and Logarithmic Functions

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Mathematics
Clarification for Topic A3.2: Exponential and Logarithmic Functions

Strand: A-Algebra

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STANDARD: A3 – FAMILIES OF FUNCTIONS

Students study the symbolic and graphical forms of each function family. By recognizing the unique characteristics of each family, students can use them as tools for solving problems or for modeling real-world situations.

Topic A3.2 Exponential and Logarithmic Functions

HSCE: A3.2.1 Write the symbolic form and sketch the graph of an exponential function given appropriate information.

Clarification: none

HSCE: A3.2.2 Interpret the symbolic forms and recognize the graphs of logarithmic functions.

Clarification: e.g., f(x) = log x, f(x) = ln x. Interpret means, for example; know that $100\ (1.02)^t$ is an exponential function with initial value 100, base 1.02, and growth rate 2%.

HSCE: A3.2.3 Apply properties of exponential and logarithmic functions.

Clarification: e.g., $a^{x+y} = a^x a^y$; log(*ab*)= log *a* + log *b*

HSCE: A3.2.4 Understand and use the fact that the base of an exponential function determines whether the function increases or decreases and how base affects the rate of growth or decay.

Clarification: none

HSCE: A3.2.5 Relate exponential functions to real phenomena, including half-life and doubling time.

Clarification: none

Background Information, Tools, and Representations

- Use graphing technology to observe the behavior of exponential functions
- Properties of Exponents and Logarithms

Exponents	Logarithms
(All laws apply for any positive a, b, x, and y.)	
$x = b^y$ is the same as $y = \log_b x$	
$b^0 = 1$	$\log_b 1 = 0$
$b^1 = b$	$\log_b b = 1$
$b^{(\log_b x)} = x$	$\log_b b^x = x$
$b^x\, b^y = b^{x+y}$	$\log_b(xy) = \log_b x + \log_b y$
$b^x \div b^y = b^{x-y}$	$\log_b(x/y) = \log_b x - \log_b y$
$(b^x)^y = b^{xy}$	$\log_b(x^y) = y \log_b x$

- See Table of Exponential and Linear Functions at the end of this document.

Assessable Content

- Incorporate concepts from A1 and A2 into assessments of this topic whenever possible.
- Assessments of this topic should include at least one situation where students are required to use exponential and/or logarithmic functions to model real-world situations.

Resources

Explore graphs of log functions.
http://purplemath.com/modules/graphlog.htm

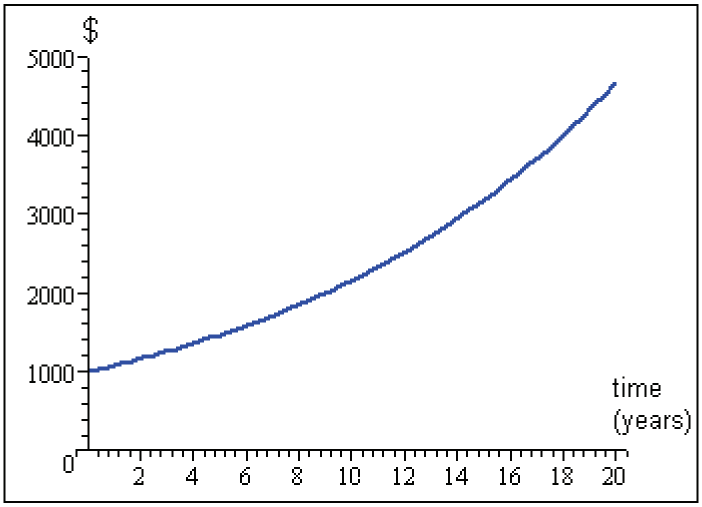
Illustration of how changes in the base a will affect the graphs of $f(x)=a^x$ and $g(x)= \log_a x$.
http://orion.math.iastate.edu/algebra/sp/xcurrent/applets/explog.html

Investigate exponential growth and decay using a spreadsheet or graphing calculator with this MI Dept of Ed Technology Enhanced Lesson Plan.
http://www.google.com/search?client=safari&rls=en&q=paper+folding+lab,+exponential&ie=UTF-8&oe=UTF-8

Clarifying Examples and Activities

HSCE: A3.2.1
Write the symbolic form and sketch the graph of an exponential function given appropriate information (e.g., given an initial value of 4 and a rate of growth of 1.5, write $f(x) = 4\ (1.5)^{x}$).

Example 1
Students will use graphing technology to observe the behavior of the exponential function, $f(x) = b^{x}$ and $f(x) = C \bullet b^{x}$



In this real world application, students will consider an investment of $1000 compounded annually at 8% for 20 years. Students must complete a journal entry with a detailed explanation of the following:

Describe the rate of growth of your investment.
Write the y= rule that describes (years, amount of money in the account)
Write the recursive rule (now-next) rule.
How is the growth rate reflected in each rule?
How long will you have to wait to double your money?
How much money will you have at the end of 5 years?

Example 2
Model the above problem using a spreadsheet program.
Assume the scenario above, except now add an extra $500 per year.
Assume the scenario above, except now subtract $100 per year.

Example 3
Suppose a single bacterium lands on an open wound. It begins spreading an infection by growing and splitting into two bacteria every 20 minutes.

- Make a table showing the number of bacteria after each 20-minute period; include at least 15 data points.
- Plot the (# of time periods, bacteria count) data points. Describe the pattern of growth.
- Write a "*y=*" rule and a recursive rule that models the situation.
- Make a prediction.

You may use a spreadsheet to explore this problem.

HSCE: A3.2.2
Interpret the symbolic forms and recognize the graphs of exponential and logarithmic functions (e.g., $f(x) = 10^{x}$, $f(x) = \log x$, $f(x) = e^{x}$, $f(x) = \ln x$)

Example 1
Explore graphs of log and ln functions. Define domain, range, symmetries, key features, intercepts, asymptotes, and rate of change.

Example 2
Consider the general exponential function $f(x)=a^x$ and its inverse logarithmic function $g(x)=\log_a x$. The applet below illustrates how changes in the base a will affect the graphs of these two functions. The red graph is $f(x)=a^x$, and the green graph is $g(x)= \log_a x$. Use the slider to explore the effect of changing the value of the base a. Note that for every a, the graphs of f and g are reflections in the line y=x, which is shown on the graph in blue.
http://orion.math.iastate.edu/algebra/sp/xcurrent/applets/explog.html

HSCE: A3.2.3
Apply properties of exponential and logarithmic functions.

Example 1
Enter both expressions into the home screen of the calculator. Are they equivalent? Explain. Explore other log properties in the same manner.

log (36 x 216)
log 36 + log 216

Example 2
Solve for x: $216 = (10)^x$

Using the graphing calculator, students can solve $y = (10)^x$ by graphing and tracing to find where $y = 216$.

Using definition of log, $216 = (10)^x$ is equivalent to $x = \log 216$
Use the LOG function of the calculator to compute.

Solve for x: $216 = e^x$.
Using the definition of natural log, $216 = e^x$ is equivalent to x=ln 216.
Use the LN function of the calculator to compute.

HSCE: A3.2.4
Understand and use the fact that the base of an exponential function determines whether the function increases or decreases and how base affects the rate of growth or decay.

Example 1
Graph two exponential functions $y = a^x$ with the following *a* values.
Let *f(x)* = an exponential function with *a* > 1.
Let *g(x)* = an exponential function with 0 < *a* < 1.

Explore the following attributes using graphs and tables:

- The domain is all real numbers R
- The *y*-intercepts
- The range is the set of positive real numbers
- The function is increasing if *a* > 1 and decreasing if 0 < *a* < 1
- The *x*-axis is a horizontal asymptote

Use the interactive feature of the graphing calculator to explore the shape of the graph as the parameter *a* changes.

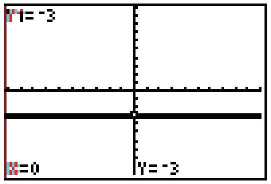
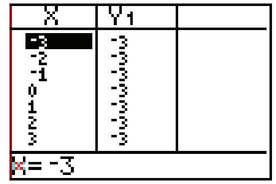
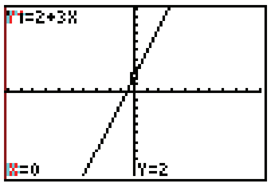
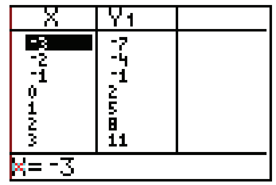
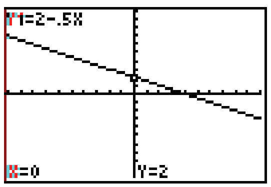
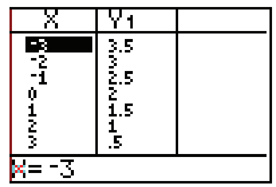
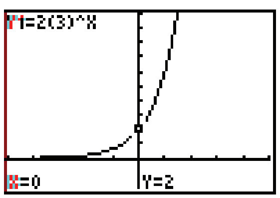
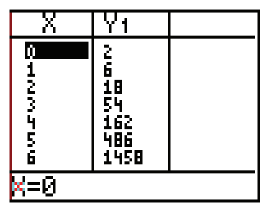
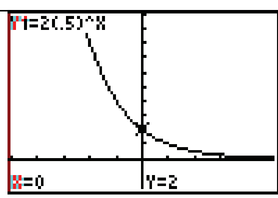
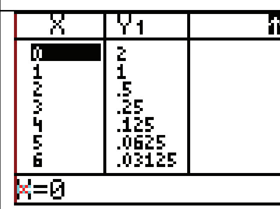
HSCE: A3.2.5
Relate exponential functions to real phenomena, including half-life and doubling time.

Example 1
The half-life of a quantity, subject to exponential decay, is the time required for the quantity to decay to half of its initial value. This activity is a simulation of a situation of half life.

Give each student a penny. Students will flip the pennies simultaneously.
When their penny turns up heads, students will be considered radioactive and must sit down. Otherwise, they must remain standing. Each time they are instructed to flip, a half-life will have passed. After each half-life, count the number of people still standing. Make a table of stage number and students standing. Continue the experiment until only one or two students remain (about 6 half-lives).
Plot the data using a scatterplot or histogram. Discuss what happens to the number of students still standing. Lead into a discussion of exponential decay and half-life.

Follow-up Questions:
1. What is happening to the number of students standing after two or three half-lives?
2. Does the same number of students sit down each time we flip?
3. About how many students would have had to sit down if we started with twice as many people?
4. What does that tell you about how the quantity of "radioactive people" affects the number that decay.
5. What is the "y=" equation and recursive rule for the experimental data of our class? What is the theoretical rule?
6. Do this simulation for 100 or 200 people using the graphing calculator random number generator. Pool the class results. Are we closer to the expected values of the simulation?
7. Do the same experiment with a 6-sided die. If you roll a 6, you are radioactive and have to sit down. Collect data. What is the half-life of your class for this experiment?
8. Model a situation with pennies where the quantity doubles at each stage.

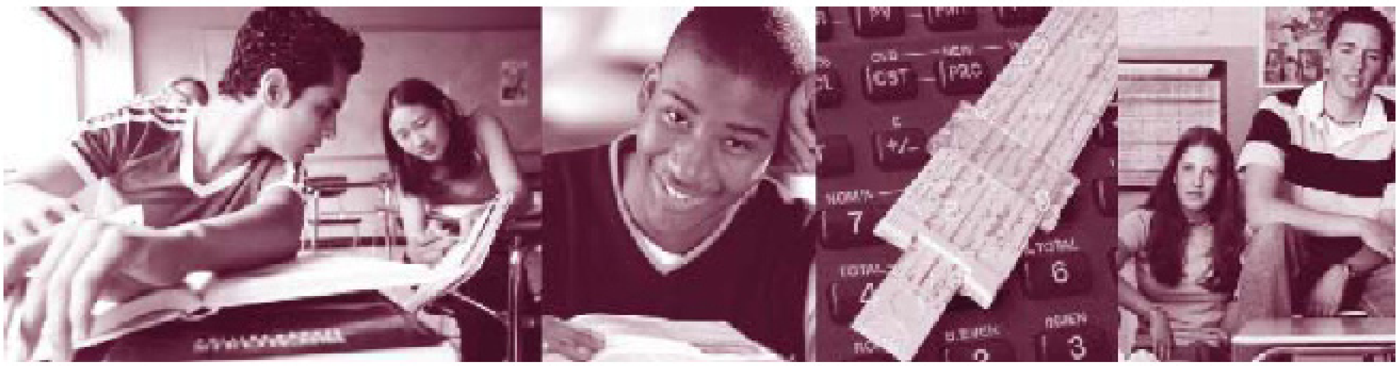
Exponential and Linear Functions

	Graph	Rate of Change in *y* values	Table	*x*-intercept	y-intercept	Domain *x* values	Range *y* values
Linear *y*=*a*+*bx* slope of *b*=0 constant function *y*=-3		Slope of zero Next = now+0 starting at -3 *(see A2.1 for explanation of now-next recursive rule)*		none	(0,*a*)	All real numbers	*y*=*a*
Linear *y*=*a*+*bx* slope of *b*>0 linear increasing *y*=2+3*x*		Increasing at a constant rate of 3 units Next = now+3 starting at 2		$\left(-\frac{a}{b},0\right)$	(0,*a*)	All real numbers	All real numbers
Linear y=a+bx slope of *b*<0 linear decreasing *y*=2-.5*x*		Decreasing at a constant rate of -.5 units Next = now + -.5 starting at 2		$\left(-\frac{a}{b},0\right)$	(0,*a*)	All real numbers	All real Numbers
Exponential Growth $y = ab^x$ $b > 1$ *Growth factor b $y = 2(3)^x$		Increasing at an increasing rate of 300% Next=now*3 starting at 2		None *x*-axis is asymptote	(0,*a*)	All real numbers	*y*>0
Exponential Decay $y = ab^x$ $0 < b < 1$ *Decay Factor *b* $y = 2(.5)^x$		Decreasing at a decreasing rate of 50% Next = now*.5 starting at 2		None x-axis is asymptote	(0,a)	All real numbers	*y*>0

Michigan Department of Education
Betty Underwood, Interim Director
Office of School Improvement
www.michigan.gov/mde

High School Content Expectations

Clarification Document



MATHEMATICS

Algebra

Topic A3.3: Quadratic Functions

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Mathematics Clarification Document
Introduction

The Michigan Mathematics High School Content Expectations (HSCE) establish what every student is expected to know and be able to do by the end of high school and outline the parameters for receiving high school credit as recently mandated by the Michigan Merit Curriculum (MMC) legislation in the state of Michigan. In an effort to support mathematics teachers as they implement the Mathematic HSCE for all students, the Michigan Mathematics Leadership Academy (MMLA), a collaboration of the Michigan Mathematics and Science Center Network, the Michigan Council of Teachers of Education (MCTM) and the Michigan Department of Education, has developed this series of companion documents, with the goal of developing a common understanding of what the expectations mean. It is intended as a tool for mathematics educators and assessment developers to provide a clearer understanding of the breadth and depth of the HSCE.

Each document in the series represents one topic of the Mathematics HSCE and follows the same format:

- Information on the strand and standard where the topic is found. This information comes directly out of the HSCE document itself but may also include some further clarifying information.
- A list of the expectations in the topic including clarifying statements where needed.
- Background Information, Tools and Representations – a reference area that includes important formulas, properties, techniques and pedagogical tips for the topic in general.
- Assessable content intended as a technical guide for assessment developers, in addition to the information found in the rest of the document.
- A resource section that provides links to websites related to the topic.
- Clarifying Examples and Activities –include brief descriptions of classroom activities, assessment items and open-ended tasks all with the intent of providing further insights into the breadth and depth of each of the expectations. In many cases, suggested interventions for struggling students are provided.

Mathematics
Clarification for Topic A3.3: Quadratic Functions

Strand: A-Algebra

In the middle grades, students see the progressive generalization of arithmetic to algebra. They learn symbolic manipulation skills and use them to solve equations. They study simple forms of elementary polynomial functions such as linear, quadratic, and power functions as represented by tables, graphs, symbols, and verbal descriptions.

In high school, students continue to develop their "symbol sense" by examining expressions, equations, and functions, and applying algebraic properties to solve equations. They construct a conceptual framework for analyzing any function and, using this framework, they revisit the functions they have studied before in greater depth. By the end of high school, their catalog of functions will encompass linear, quadratic, polynomial, rational, power, exponential, logarithmic, and trigonometric functions. They will be able to reason about functions and their properties and solve multi-step problems that involve both functions and equation-solving. Students will use deductive reasoning to justify algebraic processes as they solve equations and inequalities, as well as when transforming expressions.

This rich learning experience in Algebra will provide opportunities for students to understand both its structure and its applicability to solving real-world problems. Students will view algebra as a tool for analyzing and describing mathematical relationships, and for modeling problems that come from the workplace, the sciences, technology, engineering, and mathematics.

STANDARD: A3 – FAMILIES OF FUNCTIONS

Students study the symbolic and graphical forms of each function family. By recognizing the unique characteristics of each family, they can use them as tools for solving problems or for modeling real-world situations.

Topic A3.3 Quadratic Functions

HSCE: A3.3.1 Write the symbolic form and sketch the graph of a quadratic function given appropriate information.

Clarification: e.g., vertex, intercepts, etc.

HSCE: A3.3.2 Identify the elements of a parabola (vertex, axis of symmetry, direction of opening) given its symbolic form or its graph, and relate these elements to the coefficient(s) of the symbolic form of the function.

Clarification: none

HSCE: A3.3.3 Convert quadratic functions from standard to vertex form by completing the square.

Clarification: none

HSCE: A3.3.4 Relate the number of real solutions of a quadratic equation to the graph of the associated quadratic function.

Clarification: The real solutions are the input values, which result in an output value of zero,

HSCE: A3.3.5 Express quadratic functions in vertex form to identify their maxima or minima, and in factored form to identify their zeros.

Clarification: none

Background Information, Tools, and Representatives

- Use graphing technology to observe the behavior of quadratic functions
- Use algebra tiles/blocks to develop understanding of completing the square
- A quadratic function can be expressed in three **formats**:
  $f(x) = ax^2 + bx + c$ - general form or polynomial form (called **standard form** in HSCE); Ex. Projectile motion
  $f(x) = a(x - r_1)(x - r_2)$ is called the **factored form**, where r_1 and r_2 are the roots of the quadratic equation; Ex. Length and width of rectangle with varying dimensions
  $f(x) = a(x- h)^2 + k$ is called the **vertex form.**
- Regardless of the format, the **graph** of a quadratic function is a parabola.
  If a > 0, the parabola opens upward.
  If a < 0, the parabola opens downward.
- The **coefficient** *a* controls the speed of increase (or decrease) of the quadratic function from the vertex, bigger positive *a* makes the function increase faster and the graph appear more closed.
- The coefficients *b* and *a* control the axis of symmetry of the parabola (x coordinate of the vertex).
- The coefficient *c* controls the height of the parabola, more specifically; it is the point where the parabola crosses the y-axis.
- The **vertex** of a parabola is called the turning point. If the quadratic function is in vertex form, the vertex is (h, k). If the quadratic function is in factored form $f(x) = a(x - r_1)(x- r_2)$, the average of the two roots, $r_1 + r_2/2$ is the *x* coordinate of the vertex.
  The vertex is also the maximum point if $a < 0$ or the minimum point if a > 0.
  The vertical $x = h = -\frac{b}{2a}$ that passes through the vertex is also the axis of symmetry of the parabola.

Assessable Content

- Incorporate concepts from A1 and A2 into assessments of this topic whenever possible.
- Assessments of this topic should include at least one situation where students are required to use quadratic functions to model real-world situations.

Resources

More information on quadratic functions can be found at
http://en.wikipedia.org/wiki/Quadratic_functions

Transforming one function to another
This web activity allows students to practice graphing transformations of quadratic equations.
http://www.cet.ac.il/math/function/english/square/copying/copy4.htm

Completing the square
http://www.regentsprep.org/Regents/math/algtrig/ATE12/completesq.htm

Clarifying Examples and Activities

HSCE: A3.3.1
Write the symbolic form and sketch the graph of a quadratic function given appropriate information.

Example 1
Identify transformation techniques that move the graph of a function. For example,

- Using a parabola and the graph of its parent (basic) function, $y = x^2$, describe the transformation of the image (parabola) from the pre-image (parent function) in precise mathematical terms (e.g., reflection over the x-axis, translation 7 units to the right and 3 units down, wider, narrower).
- Discuss the relationships between the equations of the image and pre-image and the transformation of the graph.
- Generalize observations.
- Identify isometrics (rigid motions) and size transformations using coordinate maps.
- The ultimate goal is for students to identify the transformation and describe the effect(s) on the object of varying a, h, or k in the vertex form of a quadratic function $y = a(x - h)^2 + k$, or a, b or c in the standard form $y = a x^2 + bx + c$.

(From MiCLiMB)

HSCE: A3.3.2
Identify the elements of a parabola (vertex, axis of symmetry, and direction of opening) given its symbolic form or its graph and relate these elements to the coefficient(s) of the symbolic form of the function.

Example 1
Use the parent function $y = x^2$ to generate at least three variations of this function (i.e. $y = ax^2$). Graph them and summarize your findings:

- Identify important points related to the location of each parabola and state their coordinates (include x and y intercepts).
- Describe changes in location of the parabolas using these important points.
- Discuss the findings in small groups or in writing. Prompts could be these:
    - Do all parabolas have the same vertex? Why or why not?
    - Are parabolas always symmetric? Where is their axis of symmetry?
    - What determines the location of a parabola?
    - Do all parabolas open in the same direction? Why or why not?
    - What are the connections between the function and its graph?

Example 2
Given a graph of a quadratic function $y = ax^2 + bx + c$, write everything you can conclude about the value of *a* based on the graph. Discuss whether the parabola opens up or down and how it relates to the sign of *a*. Also, discuss the steepness (rate of change) of the parabola and the size of *a*.

(From MiCLiMB)

Suggestions for Differentiation

Intervention
Hanging Chains Activity, NCTM web site
Both ends of a small chain will be attached to a board with a grid on it to (roughly) form a parabola. Students will choose three points along the curve and use them to identify an equation. Repeating the process, students will discover how the equation changes when the chain is shifted.
http://illuminations.nctm.org/LessonDetail.aspx?id=L628

HSCE: A3.3.3
Convert quadratic functions from standard to vertex form by completing the square.

Example 1

Model how to geometrically complete the square with Algebra tiles.

NCTM's Illuminations website has an online instructional proof justifying the first step in completing the square. The act of "completing the square" involves taking half the coefficient of *x* in the quadratic $x^2 + ax$ and adding its square. But many students do not understand why this process works. This interactive geometric proof shows why $x^2 + ax = (x + a/2)^2 - (a/2)^2$?
http://illuminations.nctm.org/ActivityDetail.aspx?ID=132

Suggestions for differentiation

Quadratic Equations converted to vertex form may be solved using arrow diagrams:

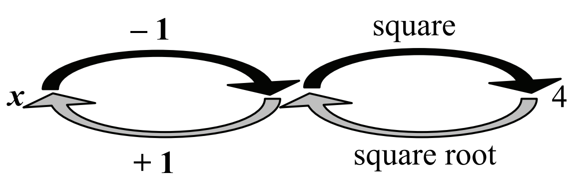
- Solve $x^2 - 2x = 3$. Complete the square; write in vertex form:

  $x^2 - 2x = 3$

  $x^2 - 2x + 1 = 3 + 1$

  $(x-1)^2 = 4$

- Draw an arrow diagram and use it to solve. The top arrows show the order of operations of the equation starting with x and ending at 4.



- The bottom arrows show the inverse operations as you work backwards starting with 4. Working backwards starting with 4, $\sqrt{4} = \pm 2$ then $\pm 2 + 1$ which yields **x=3 or x=-1**

HSCE: A3.2.4

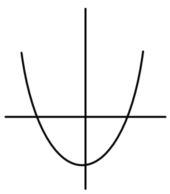
Relate the number of real solutions of a quadratic equation to the graph of the associated quadratic function.

See A2.8.1 – A2.8.3 Polynomial Functions: for related examples with zeros of cubic, quartic, and quintic functions.

Example 1

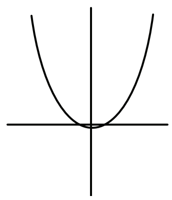
How many real solutions exist for the functions graphed below?

$y=x^2-4$



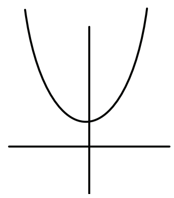
(2 real solutions)

$y=x^2$



(1 real solution)

$y=x^2+2$



(0 real solutions)

(From MiCLiMB)

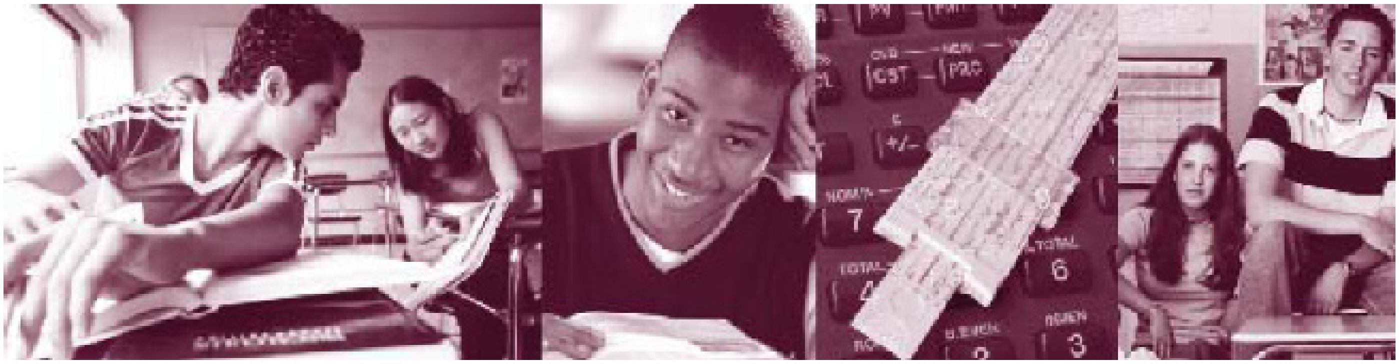
HSCE: A 3.2.5
Express quadratic functions in vertex form to identify their maxima or minima and in factored form to identify their zeros.

Example 1
Bouncing Ball: In this experiment, students collect the height versus time data of a bouncing ball using the CBR 2™. This activity investigates the values of height, time, and the coefficient A in the quadratic equation, $Y = A (X - H)^2 + K$, which describes the behavior of a bouncing ball. They graph scatter plots, graph and interpret a quadratic function, apply the vertex form of a quadratic equation, and calculate the maximum value of a parabola.
http://education.ti.com/educationportal/activityexchange/Activity.do?cid=US&aId=6493

Michigan Department of Education
Betty Underwood, Interim Director
Office of School Improvement
www.michigan.gov/mde

High School Content Expectations

Clarification Document



MATHEMATICS

Algebra

Topic A3.4: Power Functions

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Michigan State Board of Education

Mathematics Clarification Document
Introduction

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Mathematics
Clarification for Topic A3.4: Power Functions

Strand: A-Algebra

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STANDARD: A3 – FAMILIES OF FUNCTIONS

Students study the symbolic and graphical forms of each function family. By recognizing the unique characteristics of each family, students can use them as tools for solving problems or for modeling real-world situations.

Topic A3.4 Power Functions

Power functions (including roots, cubics, quartics, etc.) are functions of the form $f(x) = kx^p$ where k and p are constants. This includes, for example, integer power functions such as $f(x) = 3x^5$, root functions such as $g(x) = 4x^{\frac{1}{3}} = 4\sqrt[3]{x}$ and reciprocal functions such as $h(x) = 3x^{-2} = \frac{3}{x^2}$.

HSCE: A3.4.1 Write the symbolic form and sketch the graph of power functions.

Clarification: none

HSCE: A3.4.2 Express directly and inversely proportional relationships as functions and recognize their characteristics.

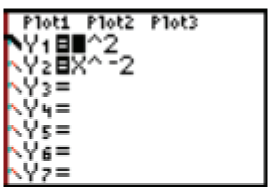
Clarification: proportional relationships: $y = kx^n$ and $y = kx^{-n}$, $n > 0$; e.g., in $y = x^3$, note that doubling x results in multiplying y by a factor of 8.

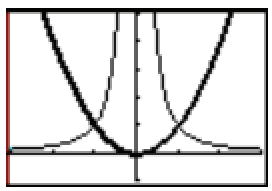
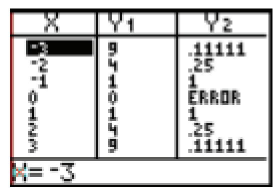
HSCE: A3.4.3 Analyze the graphs of power functions, noting reflectional or rotational symmetry.

Clarification: Students should note whether a power function is symmetric to the y-axis (even) or symmetric to the origin (odd).

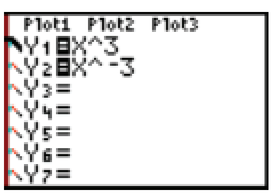
Background Information, Tools, and Representations

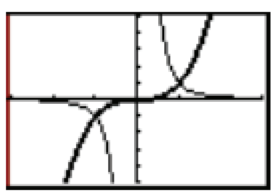
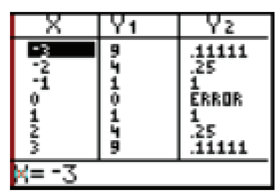
- It is easy to confuse power functions with exponential functions. Both have a basic form that is given by **two parameters**. Both forms look very similar. In exponential functions, a fixed base is raised to a variable exponent: ab^x. In power functions, however, a variable base is raised to a fixed exponent: ax^b. The parameter ***a*** serves as a simple **scaling factor**. The parameter ***b***, called either the **exponent** or the power, determines the function's rate of growth or decay. Depending on whether it is positive or negative, a whole number or a fraction, *b* will also determine the function's overall shape and behavior.
- Think about powers in terms of growth (see Clarifying Examples and Activities for HSCE A3.4.1**)**
- A monomial (a polynomial with one term – see Topic A3.5) is one subset of power functions (see also graphic at the end of this document)
- An equation in the form $y=kx$ is a direct variation. The quantities represented by x and y are directly proportional and k is the constant of variation
- An equation in the form $y=k/x$ is an inverse variation. Quantities represented by x and y are inversely proportional, and k is the constant of variation. (Also written as $y=kx^{-1}$)
- The function ax^b has reflection symmetry across the y-axis when b is an even integer. Any (x,y) on the graph/table can be paired with $(–x,y)$



- The function ax^b has 180-degree rotational symmetry about the origin when b is an odd integer. Any (x,y) on the graph/table can be paired with $(-x,-y)$



Assessable Content

- Incorporate concepts from A1 and A2 into assessments of this topic whenever possible.
- Assessments of this topic should include at least one situation where students are required to use power functions to model real-world situations.

Resources

Clarifying Examples and Activities

HSCE: A3.4.1
Write the symbolic form and sketch the graph of power functions.

Example 1
Using a graphing utility, have students compare and contrast the differences and similarities in the patterns generated in tables, graphs, and symbols of the following:

ax^2 and ax^3

x^2, x^3, x^4, x^5 ....

x^{-1}, x^{-2}, x^{-3} ....

x^2, 2x, and 2^x

Example 2
Without using a calculator, match the tables with their corresponding functions. Explain the reasoning used in each case.

X	Y1	Y2
-3	27	50
-2	12	100
-1	3	200
0	0	400
1	3	800
2	12	1600
3	27	3200

X=-3

X	Y3	Y4
-3	-3	16
-2	-1	8
-1	1	4
0	3	2
1	5	1
2	7	.5
3	9	.25

Y_4=16

$f(x) = 3 + 2x$

$f(x) = 400 * 2^x$

$f(x) = 2 * 0.5x$

$f(x) = 3x^2$

Example 3

Re-express $y = x^{\frac{1}{2}}$, $y = x^{\frac{1}{3}}$ in radical form.
Answer: ($y = \sqrt{x}$ square root function and $y = \sqrt[3]{x}$ cube root function)

Why does the square root function have a restricted domain ($x \geq 0$) and the cube root function has an unrestricted domain (all real numbers)?
Graph and discuss.

HSCE: A3.4.2
Express directly and inversely proportional relationships as functions (e.g., $y = kx^{n}$ and $y = kx^{-n}$, $n > 0$) and recognize their characteristics (e.g., in $y = x^{3}$, note that doubling x results in multiplying y by a factor of 8).

Example 1:

Explore a direct variation situation, then re-express as an inverse situation. The quantities in each situation co-vary in a unique way. In each situation, one of three quantities is a constant. This quantity is called the constant of proportionality (*k*).

	Direct Variation	**Inverse Variation**
Travel	Distance =speed*time $d = s * t$ Distance traveled is a function of how long you have been traveling *k= speed*	Time=distance/speed $t = \frac{d}{s}$ Time needed to travel a fixed distance is a function of how fast you travel *k=distance*
Earning Money	Pay=Hours worked *rate of pay $p = t * r$ Total pay earned is a function of the amount of hours worked *k= rate of pay*	Hours worked =total pay/rate of pay $t = \frac{p}{r}$ Time (hours) needed to earn a fixed amount of money is a function of how much you get paid per hour *k=total pay*
Using List Functions	Enter data in (list 1, list 2) that is directly proportional. List 1/list 2=*k* (slope)	Enter data in (list 1, list 2) that is inversely proportional List 1*list 2=*k*

Example 2

Growing Cubes Investigation: Students discover the principle of proportionality

Build or draw a set of cubes with the following edge lengths. Be sure to identify the measure used.

Edge Length **unit =**	**Perimeter of one face** **unit =**	**Surface Area of the cube** **unit =**	**Volume of Cube** **unit =**
1			
2			
3			
4			
10			
k			

Describe the patterns in the tables, graphs, and equations which relate edge length to perimeter, edge length to surface area and edge length to volume of a cube.

When edge length is increased by a factor of *k*, how does and volume and surface area increase? Explain.

HSCE: A3.4.3
Write the symbolic form and sketch the graph of power functions.

Example 1

Explore the symmetry patterns of even and odd power functions using tables and graphs. Write a compare/contrast summary of the even and odd functions. Include symmetry patterns, domain and range, key features, asymptotes, possible shapes, end behaviors, and rates of change.

Power Function Family Tree*

One subset of power functions are monomials (polynomials with one term).

Power
$f(x)=kx^p$ where k and p are any real constant

Monomial
$f(x)= kx^p$ where p is any non-negative integer

Quadratic
of the form $f(x)= kx^2$

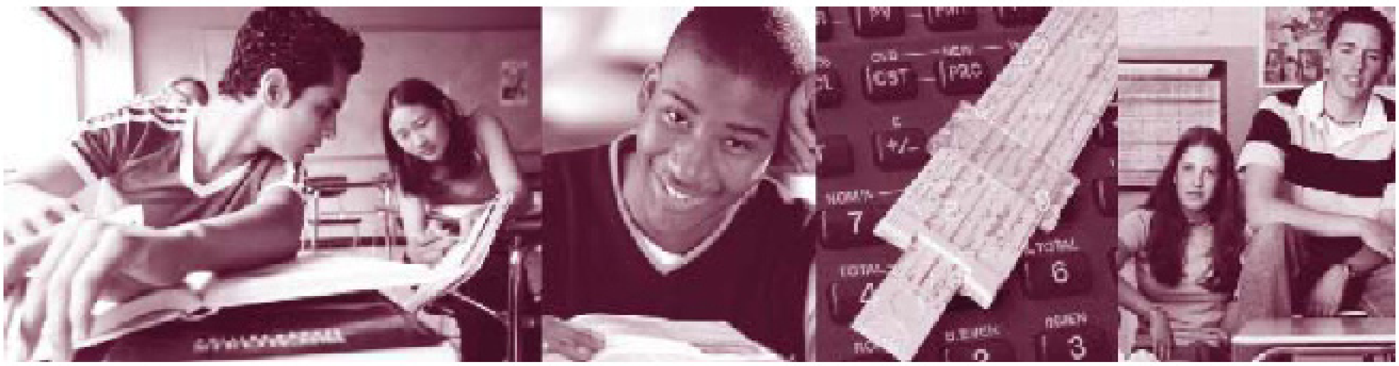
Linear
of the form $f(x)= kx$

* Based on the HSCE function families

Michigan Department of Education
Betty Underwood, Interim Director
Office of School Improvement
www.michigan.gov/mde

High School Content Expectations

Clarification Document



MATHEMATICS

Quantitative Literacy
Topic L1.1: Number Systems
and Number Sense

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Mathematics Clarification Document
Introduction

The Michigan Mathematics High School Content Expectations (HSCE) establish what every student is expected to know and be able to do by the end of high school and outline the parameters for receiving high school credit as recently mandated by the Michigan Merit Curriculum (MMC) legislation in the state of Michigan. In an effort to support mathematics teachers as they implement the Mathematic HSCE for all students, the Michigan Mathematics Leadership Academy (MMLA), a collaboration of the Michigan Mathematics and Science Center Network, the Michigan Council of Teachers of Education (MCTM) and the Michigan Department of Education, have developed this series of companion documents, with the goal of developing a common understanding of what the expectations mean. It is intended as a tool for mathematics educators and assessment developers to provide a clearer understanding of the breadth and depth of the HSCE.

Each document in the series represents one topic of the Mathematics HSCE and follows the same format:

- Information on the strand and standard where the topic is found. This information comes directly out of the HSCE document itself but may also include some further clarifying information.
- A list of the expectations in the topic including clarifying statements where needed.
- Background Information, Tools and Representations – a reference area that includes important formulas, properties, techniques and pedagogical tips for the topic in general.
- Assessable content intended as a technical guide for assessment developers, in addition to the information found in the rest of the document.
- A resource section that provides links to websites related to the topic.
- Clarifying Examples and Activities –include brief descriptions of classroom activities, assessment items and open-ended tasks all with the intent of providing further insights into the breadth and depth of each of the expectations. In many cases, suggested interventions for struggling students are provided.

Mathematics
Clarification for Topic L1.1: Number Systems and Number Sense

STRAND: L-QUANTITATIVE LITERACY

"In an increasingly complex world, adults are challenged to apply sophisticated quantitative knowledge and reasoning in their professional and personal lives. The technological demands of the workplace, the abundance of data in the political and public policy context, and the array of information involved in making personal and family decisions of all types necessitate an unprecedented facility not only with fundamental mathematical, statistical, and computing ideas and processes, but with higher-order abilities to apply and integrate those ideas and processes in a range of areas." [1]

The Michigan Grade Level Content Expectations in Mathematics for grades K-8 prescribe a thorough treatment of number, including strong emphasis on computational fluency and understanding of number concepts, to be completed largely by the sixth grade. The expectations in this Quantitative Literacy and Logic strand provide a definition of secondary school quantitative literacy for all students and emphasize the importance of logic as part of mathematics and in everyday life. They assume fluency (that is, efficiency and accuracy) in calculation with the basic number operations involving rational numbers in all forms (including percentages and decimals), without calculators.

Mathematical reasoning and logic are at the heart of the study of mathematics. As students progress through elementary and middle school, they increasingly are asked to explain and justify the thinking underlying their work. In high school, students peel away the contexts and study the language and thought patterns of formal mathematical reasoning. By learning logic and by constructing arguments and proofs, students will strengthen not only their knowledge and facility with mathematics, but also their ways of thinking in other areas of study and in their daily lives.

Connections and applications of number ideas and logic to other areas of mathematics, such as algebra, geometry, and statistics, are emphasized in this strand. Number representations and properties extend from the rational numbers into the real and complex numbers, as well as to other systems that students will encounter both in the workplace and in more advanced mathematics. The expectations for calculation, algorithms and estimation reflect important uses of number in a range of real-life situations. Ideas about measurement and precision tie closely to geometry.

STANDARD: L1: REASONING ABOUT NUMBERS, SYSTEMS, AND QUANTITATIVE SITUATIONS

Based on their knowledge of the properties of arithmetic, students understand and reason about numbers, number systems, and the relationships between them. They represent quantitative relationships using mathematical symbols, and interpret relationships from those representations.

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[1] Estry, D., & Ferrini-Mundy, J. (January, 2005). Quantitative Literacy Task Force Final Report and Recommendations. East Lansing: Michigan State University.

TOPIC L1.1 NUMBER SYSTEMS AND NUMBER SENSE

HSCE: L1.1.1 Know the different properties that hold in different number systems, and recognize that the applicable properties change in the transition from the positive integers, to all integers, to the rational numbers, and to the real numbers.

Clarification: The properties of the real number system allow expressions to be rewritten in an equivalent form to represent a particular situation, and then to work within that representation free from context. For instance, the distributive property allows factoring of algebraic expressions. (See also A1.1.1, A1.1.3 and A1.2.2).

Students should know which number properties hold (are true) for each number system and vice versa.

HSCE: L1.1.2 Explain why the multiplicative inverse of a number has the same sign as the number, while the additive inverse of a number has the opposite sign.

Clarification: This ties the ideas of multiplicative inverse and additive inverse to the rules of signs for multiplying signed numbers and to the concept of absolute value, respectively.

HSCE: L1.1.3 Explain how the properties of associativity, commutativity, and distributivity, as well as identity and inverse elements, are used in arithmetic and algebraic calculations.

Clarification: These properties will be used in both formal proofs and informal justifications as students simplify arithmetic and algebraic expressions, and solve algebraic equations.

HSCE: L1.1.4 Describe the reasons for the different effects of multiplication by, or exponentiation of, a positive number by a number less than 0, a number between 0 and 1, and a number greater than 1.

Clarification: The idea here is that when numbers from different families are used as the exponent, taking a positive number (which could be rational, irrational, or a natural number) to that power results in very different-looking results, depending upon whether the base was whole, rational, or irrational.

HSCE: L1.1.5 Justify numerical relationships.

Clarification: Demonstrate through formal proof or through informal justification mathematical statements such as:

- the sum of even integers is even
- every integer can be written as $3m+k$, where k is 0, 1, or 2, and m is an integer
- the sum of the first n positive integers is $n\ (n+1)/2)$
- the sum of any two odd integers is even
- the sum of an even integer and an odd integer is odd
- when multiplying, an even number of negative factors yields a positive result while an odd number of negative factors yields a negative result.

HSCE: L1.1.6 Explain the importance of the irrational numbers $\sqrt{2}$ and $\sqrt{3}$ in basic right triangle trigonometry, and the importance of π because of its role in circle relationships.

Clarification: The "importance" part of the statement refers to the idea that $\sqrt{2}$ and $\sqrt{3}$ are exact values, not approximations. Their appearance in trigonometry can easily be traced back to working with special right triangles.

Pi is the ratio of the circumference of any circle to its diameter. Whether working with circumference, area, or volume, or working with radian angle measure and trigonometry, π is used to represent the exact value of this ratio. Being able to work with π rather than using and approximation in calculations is a valuable skill as one's results are more accurate. Plus, by working with π itself in calculations, one often can save much computation work.

Background Information, Tools, and Representations

- Properties include:
  - commutative property of addition (and of multiplication);
  - associative property of addition (and of multiplication);
  - distributive property of addition over multiplication;
  - additive identity property;
  - additive inverse property;
  - multiplicative identity property;
  - multiplicative inverse property;
  - closure property (under addition, subtraction, multiplication, division, exponentiation (includes roots));
  - density property;

- Number systems include natural numbers (Ν); whole numbers (Ω); integers (Ζ); rational numbers (Θ); irrational numbers (Ι)[1]; real numbers (Ρ).

- The **multiplicative inverse** of a number *x*, denoted $1/x$ or x^{-1}, is the number which, when multiplied by *x*, yields 1. The multiplicative inverse of *x* is also called the **reciprocal** of *x*. One computes the multiplicative inverse of a (real) number by dividing 1 by the number. The **additive inverse**, or **opposite**, of a number *n* is the number that, when added to *n*, yields zero. The additive inverse of *n* is denoted $-n$.

Assessable Content

- Restrict to real numbers and subsets thereof.
- L1.1.5 - Stick to straightforward numerical relationships which students are likely to see and use quite often (odd/even numbers, sign outcomes, and the like).

Resources

For more information on how properties apply to algebraic expressions:
http://www.math.com/school/subject2/lessons/S2U2L1DP.html

For more information on sets and number systems:
http://www.wtamu.edu/academic/anns/mps/math/mathlab/int_algebra/int_alg_tut3_sets.htm

Website for historical Algebraic and Geometric Proofs of even and odd numbers:
http://www.mathgym.com.au/history/pythagoras/Activity3.htm

Website for further look at even and odd numbers
http://planetmath.org/encyclopedia/OddInteger.html

[1] "Ι" as a symbol for the set of irrational numbers is not nearly as commonly used as the other number set symbols. Frequently no symbol is used for the set of irrational numbers.

Clarifying Examples and Activities

HSCE: L1.1.1

Know the different properties that hold in different number systems, and recognize that the applicable properties change in the transition from the positive integers, to all integers, to the rational numbers, and to the real numbers.

Example 1

Deanna creates a new operation and uses the symbol ♦ to represent the operation. She defines a ♦ b as $a^2 \div b$. For which number families is this operation closed?

[Rational, real, and complex (limit complex families in Algebra 1 to enrichment tasks)]

Example 2

Students can use a chart to list the various number families across the top, and the various properties they study over time along the side, and determine which properties hold for each family under a given operation. A sample chart is available at the end of this document.

Example 3

Q: Which of the number families below is closed under addition and multiplication, but is NOT closed under division?

A. complex numbers

B. rational numbers

C. natural numbers

D. real numbers

Example 4

Q. The Density Property states that between any two members of a set of numbers, there exists at least one other member of that set of numbers. To which set listed below does the density property apply?

A. integers

B. whole numbers

C. natural numbers

D. rational numbers

Suggestions for differentiation

- Enrichment activities could extend properties to the complex numbers.

HSCE: L1.1.2 Explain why the multiplicative inverse of a number has the same sign as the number, while the additive inverse of a number has the opposite sign.

Example 1

Relate to zero pairs in algebraic expressions (term used in middle school integer addition and subtraction).

A zero pair is a pair of numbers whose sum is zero (sum of a number and its additive inverse). Zero pairs are used to simplify addition and subtraction problems.

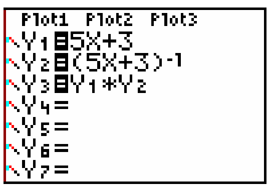
Take the following expression: 2 + 3 - 2 + 9 - 3

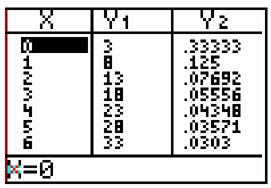
Rewrite the expression: 2 + 3 + (-2) + 9 + (-3)

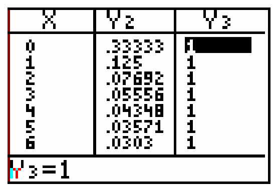
Group the zero pairs together 2 + (-2) + 9 + 3 + (-3) and get 0 + 9 + 0 which is 9.

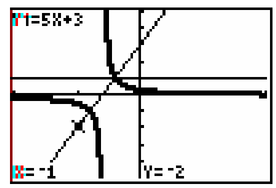
Example 2

Explore patterns in tables and graphs. Trace on the same x value on y1, y2, and y3. Describe patterns.







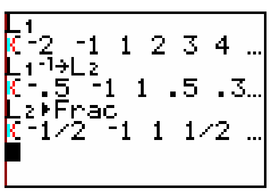


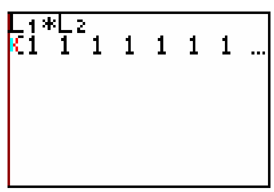
(Use with A2.2.1: Combine functions by addition, subtraction, multiplication, and division).

Suggestions for differentiation

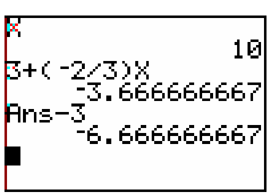
Intervention

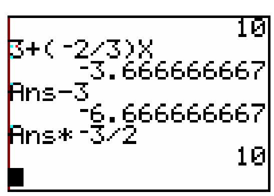
- ❖ Use the list functions of the graphing calculator to explore multiplicative inverse of negative and positive numbers (use x^{-1} button)





- ❖ Explore undoing expressions using multiplicative and additive inverse on the home screen of the graphing calculators In this case, $10 \rightarrow x$





HSCE: **L1.1.3** Explain how the properties of associativity, commutativity, and distributivity, as well as identity and inverse elements, are used in arithmetic and algebraic calculations.

Example 1

Use algebraic reasoning to explain how the historical algorithms below work.

Egyptian Multiplication
http://atozteacherstuff.com/pages/296.shtml

Lattice Multiplication
http://mathforum.org/library/drmath/view/52468.html

Napier's Bones Multiplication
http://en.wikipedia.org/wiki/Napier's_bones

Multiplying by Drawing Lines
http://www.i-am-bored.com/bored_link.cfm?link_id=20684

Example 2

Students typically learn about the concepts of identity, inverse, commutativity, and associativity by exploring the four basic operations (+, −, ×, and ÷) with integers. In this lesson, students investigate these concepts using a geometric model. Moves are performed with a rectangle, and the results of an operation that combines two moves are analyzed. Students determine if the operation is commutative or associative; if an identity element exists; or if there are inverses for any of the moves.
http://illuminations.nctm.org/LessonDetail.aspx?ID=L566

Example 3

Prove the following using the appropriate property to justify each step.

$$a + 3(b - a) = 3b - 2a$$

Suggestions for differentiation

- Write three expressions (shown below) to calculate the perimeter of a rectangle with length L and width W. Explain the thinking behind each expression using a rectangle drawing. Then explain why they are equivalent using algebraic reasoning

$$P=L+W+L+W$$
$$P=2L+2W$$
$$P=2(L+W)$$

- Visual represent the distributive property using Algebra Tiles; use to analyze algorithms in sample activities found at:
http://www.regentsprep.org/Regents/math/realnum/Tdistrib.htm

HSCE: L1.1.4 Describe the reasons for the different effects of multiplication by, or exponentiation of, a positive number by a number less than 0, a number between 0 and 1, and a number greater than 1.

Clarifying Examples and Activities:

Example 1

Compare what happens when the exponents remain the same but the base changes:

$$4^{-3} = \frac{1}{4^3} = \frac{1}{64}$$

$$4^{1/2} = \sqrt{4} = 2$$

$$4^5 = 1024$$

but starting with a base betwen 0 and 1...

$$\left(\frac{1}{4}\right)^{-3} = 4^3 = 64$$

$$\left(\frac{1}{4}\right)^{1/2} = \sqrt{1/4} = \frac{1}{2}$$

$$\left(\frac{1}{4}\right)^{5} = \frac{1}{1024}$$

The first example with 4 (a number larger than 1) as the base gives a comparatively small number: 1/64. This would be expected because we take the reciprocal of 4, giving ¼, and then cube it – multiplying a fraction between zero and one by itself, and by itself again. This yields a much smaller number that with which we started. However, when we take a number between 0 and 1 as the base, such as ¼ in the second set of examples, to the -3 power, we find it's reciprocal – which will be a number larger than 1, and then take it to the 3rd power – giving us a much larger number than the one with which we started. The other examples could be explained in a similar fashion.

HSCE: L1.1.5 Justify numerical relationships.

Example 1

Students explore, invent, and explain equivalent rules for the sum of the first *n* positive integers using square tiles.
http://www.learner.org/channel/courses/teachingmath/grades9_12/session_03/section_01_b.html

Example 2

The Game of Morra
http://en.wikipedia.org/wiki/Odd_or_even
While there are many variations, the basic game of Morra is played with two players. One of the players is designated the "odds" player, while the other is labeled "evens". Players hold one hand out in front of them and then count together to three (some people chant "Once, twice, thrice, SHOOT!" or even just "One, two, three, SHOOT!"). On three (or "shoot", whichever is agreed beforehand), both players hold out either one or two fingers. If the sum of fingers held out by both players is an even number (i.e. two or four) then the "evens" player wins. If the number of fingers sums to an odd number (i.e. three), the "odds" player is the winner. Since there are two possible ways to add up to three, both even and odd have a 50:50 chance of winning. Some variants of the game also make use of money, with the winner earning a number of dollars equal to the sum of fingers displayed. Ask students to look for a pattern of odd and even and make a conjecture about any patterns they notice.

Suggestions for differentiation

Enrichment

- ❖ Prove that every integer can be written as 3m+k, where k is 0, 1, or 2. Explore this theorem at http://en.wikipedia.org/wiki/Division_algorithm

- ❖ Historical look at Euclid’s propositions including even and odd numbers http://aleph0.clarku.edu/~djoyce/java/elements/bookIX/bookIX.html

HSCE: L1.1.6 Explain the importance of the irrational numbers $\sqrt{2}$ and $\sqrt{3}$ in basic right triangle trigonometry, and the importance of π because of its role in circle relationships.

Example 1

Find the area of a circle with radius of 7cm.

Exact value: $A = \pi r^2 = 49\pi$ square cm.
Approximate value: A=153.94 square cm. (rounded to nearest hundredth)

Example 2

Explain how to find the Transmission factor for any system of two pulleys (or sprockets).

A two pulley system has driver circumference C_{driver} and follower circumference $C_{follower}$.

Transmission Factor = $\dfrac{Circumference_{driver}}{Circumference_{follower}} = \dfrac{2\pi r_{driver}}{2\pi r_{follower}} = \dfrac{r_{driver}}{r_{follower}}$

As the driver pulley turns once, the follower pulley turns by a factor of $\dfrac{r_{driver}}{r_{follower}}$ (direct proportion).

Example 3

Students should work with a number of isosceles right triangles to see the pattern that the hypotenuse is always (leg)$\bullet\sqrt{2}$. They should then be able to prove why this works by using algebra and the Pythagorean Theorem. They can then tie this to finding exact values on the unit circle for sine, cosine, and tangent of integer multiples of 45°.

Example 4

Students should work with a variety of equilateral triangles, dropping an altitude from one vertex. Using the Pythagorean Theorem they then find that in the resulting 30°-60°-90° triangles, the hypotenuse is twice as large as the shorter leg, and the longer leg is (shorter leg)$\bullet\sqrt{3}$. This can then be tied to finding the exact values on the unit circle for sine, cosine, and tangent of integer multiples of 30° and 60°.

Example 5

Teachers may want to highlight to students how various cultures have defined π over time, and how right up to the last century, people were looking for a "shortcut" to working with π (a couple states tried to legislate its value!).

Suggestions for differentiation

Intervention

Using two cylinders of different sizes and a large rubber band (pulley belt), collect data on how much the follower pulley turns for various turns of the driver pulley. Graph data. Find line of best fit. Explain how the equation models the motion. Reverse driver and follower. How does this change the equation that explains the motion?

Enhancement

Study the role of e. As students progress through their advanced mathematics studies, they will find e showing up in even more places. Over time, students should recognize that e appears in a number of places, such as working with natural logarithms and with compound interest – seemingly disparate concepts. Like working with π, it is often easier to leave *e* as *e*, and not use an approximation (unless working within a context requiring a result to a given number of decimal places).

Table of Properties and Number Families

Number Family→ Property ↓	Natural Numbers Or Counting Numbers [1,2,3, . . .]	Whole Numbers [0,1,2,3,4,. . . .]	Integers [. . .-3,-2,-1, 0,1,2,3, . . .]	Rational Numbers [Numbers of the form $\frac{a}{b}$; *a* and *b* integers, $b \neq 0$	Irrational Numbers [Cannot be expressed as a ratio of integers]	Real Numbers [Rational & Irrationals: all points on the number line]
Commutative Property of Addition	Yes	Yes	Yes	Yes	Yes	Yes
Associative Property of Addition	Yes	Yes	Yes	Yes	Yes	Yes
Additive Identity	NO (need a zero)	Yes	Yes	Yes	Yes	Yes
Additive Inverse	NO (would need integers for this)	NO (would need integers for this)	Yes	Yes	Yes	Yes
Commutative Property of Multiplication	Yes	Yes	Yes	Yes	Yes	Yes
Associative Property of Multiplication	Yes	Yes	Yes	Yes	Yes	Yes
Multiplicative Identity	Yes	Yes	Yes	Yes	Yes	Yes
Multiplicative Inverse	NO (would need rational numbers)	NO (would need rational numbers)	NO (would need rational numbers)	Yes	Yes	Yes
Distributive Property of Multiplication over Addition	Yes	Yes	Yes	Yes	Yes	Yes
Closure under addition	Yes	Yes	Yes	Yes	Yes	Yes
Closure under subtraction	NO – (would wind up creating integers)	NO – (would wind up creating integers)	Yes	Yes	Yes	Yes
Closure under multiplication	Yes	Yes	Yes	Yes	Yes	Yes
Closure under division	NO (would wind up creating rationals)	NO (would wind up creating rationals)	NO (would wind up creating rationals)	Yes	Yes	Yes

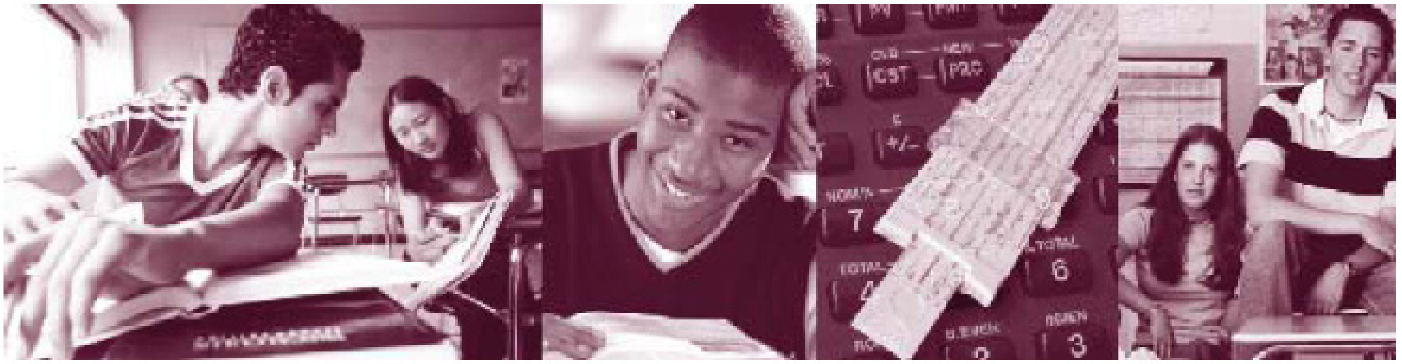
Closure under… (operation of your choice)	Depends on the operation you choose	Depends on the operation you choose	Depends on the operation you choose	Depends on the operation you choose	Depends on the operation you choose	Depends on the operation you choose
Density Property	NO (between each pair you can't find another number)	NO (between each pair you can't find another number)	NO (between each pair you can't find another number)	Yes	Yes	Yes

MICHIGAN Department of Education

Michigan Department of Education
Betty Underwood, Interim Director
Office of School Improvement
www.michigan.gov/mde

High School Content Expectations

Clarification Document



MATHEMATICS

Quantative Literacy
Topic L1.2: Representations and Relationships

Mathematics Clarification Document
Introduction

The Michigan Mathematics High School Content Expectations (HSCE) establish what every student is expected to know and be able to do by the end of high school and outline the parameters for receiving high school credit as recently mandated by the Michigan Merit Curriculum (MMC) legislation in the state of Michigan. In an effort to support mathematics teachers as they implement the Mathematic HSCE for all students, the Michigan Mathematics Leadership Academy (MMLA), a collaboration of the Michigan Mathematics and Science Center Network, the Michigan Council of Teachers of Education (MCTM) and the Michigan Department of Education, has developed this series of companion documents, with the goal of developing a common understanding of what the expectations mean. It is intended as a tool for mathematics educators and assessment developers to provide a clearer understanding of the breadth and depth of the HSCE.

Each document in the series represents one topic of the Mathematics HSCE and follows the same format:

- Information on the strand and standard where the topic is found. This information comes directly out of the HSCE document itself but may also include some further clarifying information.
- A list of the expectations in the topic including clarifying statements where needed.
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- Assessable content intended as a technical guide for assessment developers, in addition to the information found in the rest of the document.
- A resource section that provides links to websites related to the topic.
- Clarifying Examples and Activities –include brief descriptions of classroom activities, assessment items and open-ended tasks all with the intent of providing further insights into the breadth and depth of each of the expectations. In many cases, suggested interventions for struggling students are provided.

Mathematics

Clarification for Topic L1.2: Representations and Relationships

Strand: L-Quantitative Literacy

"In an increasingly complex world, adults are challenged to apply sophisticated quantitative knowledge and reasoning in their professional and personal lives. The technological demands of the workplace, the abundance of data in the political and public policy context, and the array of information involved in making personal and family decisions of all types necessitate an unprecedented facility not only with fundamental mathematical, statistical, and computing ideas and processes, but with higher-order abilities to apply and integrate those ideas and processes in a range of areas." [1]

The Michigan Grade Level Content Expectations in Mathematics for grades K-8 prescribe a thorough treatment of number, including strong emphasis on computational fluency and understanding of number concepts, to be completed largely by the sixth grade. The expectations in this Quantitative Literacy and Logic strand provide a definition of secondary school quantitative literacy for all students and emphasize the importance of logic as part of mathematics and in everyday life. They assume fluency (that is, efficiency and accuracy) in calculation with the basic number operations involving rational numbers in all forms (including percentages and decimals), without calculators.

Mathematical reasoning and logic are at the heart of the study of mathematics. As students progress through elementary and middle school, they increasingly are asked to explain and justify the thinking underlying their work. In high school, students peel away the contexts and study the language and thought patterns of formal mathematical reasoning. By learning logic and by constructing arguments and proofs, students will strengthen not only their knowledge and facility with mathematics, but also their ways of thinking in other areas of study and in their daily lives.

Connections and applications of number ideas and logic to other areas of mathematics, such as algebra, geometry, and statistics, are emphasized in this strand. Number representations and properties extend from the rational numbers into the real and complex numbers, as well as to other systems that students will encounter both in the workplace and in more advanced mathematics. The expectations for calculation, algorithms and estimation reflect important uses of number in a range of real-life situations. Ideas about measurement and precision tie closely to geometry.

Standard: L1: Reasoning about Numbers, Systems, and Quantitative Situations

> Based on their knowledge of the properties of arithmetic, students understand and reason about numbers, number systems, and the relationships between them. They represent quantitative relationships using mathematical symbols, and interpret relationships from those representations.

[1] Estry, D., & Ferrini-Mundy, J. (January, 2005). Quantitative Literacy Task Force Final Report and Recommendations. East Lansing: Michigan State University.

Topic L1.2 Representations and Relationships

HSCE: L1.2.1 Use mathematical symbols to represent quantitative relationships and situations.

Clarification: (e.g., interval notation, set notation, summation notation).

As part of learning to communicate mathematically, students need to learn how to read and use mathematical symbols appropriately. For these symbols, they should learn the name of the symbol, how to read it in context of a mathematical statement, translate the statement into an equivalent form using other symbols (if appropriate), and be able to use it appropriately in their own writing.

Students are also expected to learn to read, interpret, and write mathematical statements using interval notation, set notation, and summation notation. Students often find these notation conventions somewhat confusing, as a few symbols are used to convey a much more lengthy English statement.

HSCE: L1.2.2 Interpret representations that reflect absolute value relationships, in such contexts as error tolerance.

Clarification: $|x-a| < b$, or $a \pm b$

HSCE: L1.2.3 Use vectors to represent quantities that have magnitude and direction, interpret direction and magnitude of a vector numerically, and calculate the sum and difference of two vectors.

Clarification: none

HSCE: L1.2.4 Organize and summarize a data set in a table, plot, chart, or spreadsheet; find patterns in a display of data; understand and critique data displays in the media.

Clarification: none

Background Information, Tools, and Representations

- Over the course of their study of mathematics in high school, students will encounter a number of familiar (e.g., +, -, x, =) and unfamiliar (e.g.,!,$\in$,$\sum$) symbols. It is not practical to list all of these symbols here.
- Interval Notation:

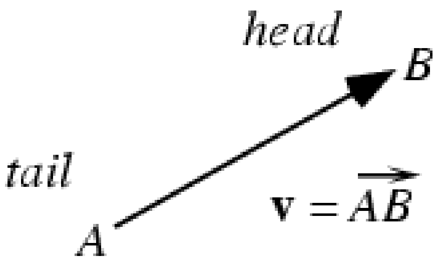
  [-6, 3] the closed interval from -6 to 3; can also be written as -6 ≤ x ≤ 3
  (-6, 3] the half-open (or half-closed) interval from -6 to 3, including 3; can also be written as -6 < x ≤ 3
  [-6, 3) the half-open (or half-closed) interval from -6 to 3, including -6; can also be written as -6 ≤ x < 3
  (-6, 3) the open interval from -6 to 3; can also be written -6 < x < 3
  Students should be able to move fluently among interval notation, compound inequality notation, set-builder notation, and the graph of an interval on a number line.
- Set Notation

  { } braces for including set members
  {1, 2, 3, 4, 5} roster notation, listing each set element/member
  element any member of a set
  $\in$ "is an element of"
  $\notin$ "is not an element of"
  {x|x<3} "the set of all x such that x is less than 3"
  {x:x<3} "the set of all x such that x is less than 3"
  N the set of all natural (counting) numbers
  Ω the set of all whole numbers
  $\mathbb{Z}$ the set of all integers
  $\mathbb{Q}$ the set of all rational numbers;
  I "I" as a symbol for the set of irrational numbers is not nearly as commonly used as the other number set symbols. Frequently no symbol is used for the set of irrational numbers.
  P the set of all real numbers
  X the set of all complex numbers
  ∅ the null set (set with no elements)
  { } the empty set (set with no elements)
  ∩ intersection of sets
  ∪ union of sets
  A, B, C… naming sets using capital block letters
- Summation Notation (Sigma Notation)

  A short way of indicating a set of numbers which are to be added is to use summation notation (also called sigma notation). Summation notation uses the Greek capital letter sigma (Σ) along with additional information provided above and below the sigma symbol to indicate what expression is being considered, the starting value for evaluating the expression, and the ending value for evaluating the expression. For example: $\sum_{k=1}^{10} k^2$ where:

  k is the index variable, showing which variable in the expression will be changing values; 1 is the *lower bound (or limit) of summation* and 10 is the *upper bound (or limit) of summation*; k^2 is the *argument* of the summation.

- ❖ Vector quantities are often represented by scaled vector diagrams.



Assessable Content

- ❖ L1.2.1 should not be tested separately. Students will be expected to know and use appropriate symbols and notation in assessment of other expectations.

Resources

Online Set Notation Activities
http://webspace.ship.edu/deensley/DiscreteMath/flash/ch3/sec3_1/setnotation.html

Students use vectors to represent displacement and velocity and then use both graphical and analytical methods to determine vector resultants and components.
http://k12science.org/curriculum/vectorproj/project_information.html

Vector Investigation: Dual Vector, Airplane Storm Chaser
http://illuminations.nctm.org/ActivityDetail.aspx?ID=43

Explore how characteristics of the vector represent movement.
http://standards.nctm.org/document/eexamples/chap7/7.1/index.htm

A study of motion will involve the introduction of a variety of quantities which are used to describe the physical world. Examples of such quantities include distance, displacement, speed, velocity, acceleration, force, mass, momentum, energy, work, power, etc.
http://www.glenbrook.k12.il.us/gbssci/phys/Class/vectors/u3l1a.html

The resolution of a vector into its components can be used in the addition and subtraction of vectors.
http://www.physics.uoguelph.ca/tutorials/vectors/vectors.html

The National Library of Virtual Manipulatives has activities for organizing and interpreting data for students in grades 9-12.
http://nlvm.usu.edu/en/nav/topic_t_5.html

Students determine the type of data which they are looking:
http://education.ti.com/educationportal/activityexchange/Activity.do?cid=US&aId=7932.

Clarifying Examples and Activities

HSCE: L1.2.1 Use mathematical symbols to represent quantitative relationships and situations.

Example 1

Express the following inequality using interval notation:
{x such that x ≤ -10 }

a) $(-\infty, -10)$
b) $(-\infty, -10]$
c) $(-10, \infty)$
d)
e) $[-10, \infty]$

HSCE: L1.2.2 Interpret representations that reflect absolute value relationships in such contexts as error tolerance.

Suggestions for differentiation

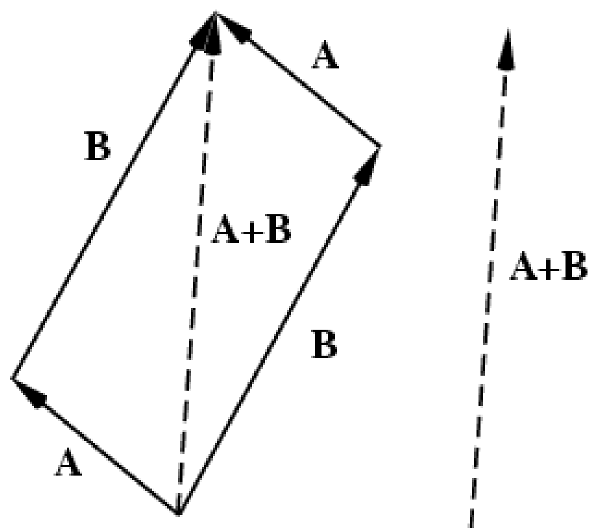
Intervention

Fill a mason jar with jelly beans. Ask students to start thinking about how many jelly beans are in the jar. During your unit on absolute value equations and inequalities, ask the students to submit a guess as to how many jelly beans are in the jar. Make a histogram or stem and leaf plot of the guesses. Ask students to write an absolute value "expression" (not equation) that can be used to determine who was closest to the actual number of jelly beans in the jar.
[http://regentsprep.org/regents/mathb/7D6/absteacher.htm]

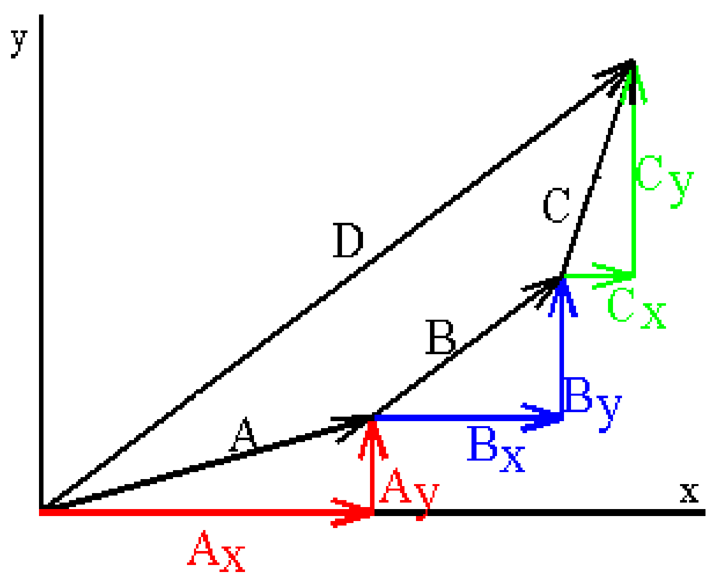
HSCE: L1.2.3 Use vectors to represent quantities that have magnitude and direction, interpret direction and magnitude of a vector numerically, and calculate the sum and difference of two vectors.

Example 1

The so-called parallelogram law gives the rule for vector addition of two or more vectors. For two vectors A and B, the vector sum A+B is obtained by placing them head to tail and drawing the vector from the free tail to the free head. This vector is called the resultant.



The resolution of a vector into its components can be used in the addition and subtraction of vectors.



HSCE: L1.2.4 Organize and summarize a data set in a table, plot, chart, or spreadsheet; find patterns in a display of data; understand and critique data displays in the media.

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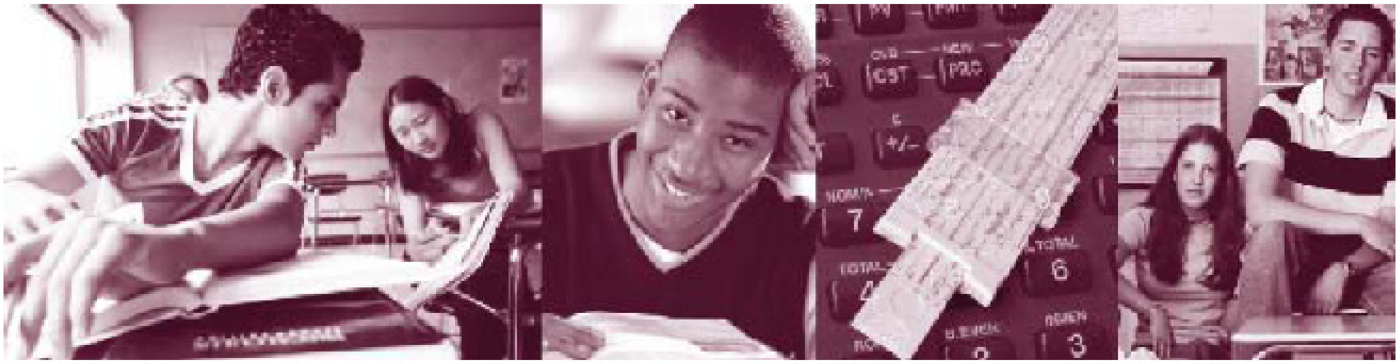
High School Content Expectations

Clarification Document



MATHEMATICS

Statistics and Probability
Standard S2: Bivariate Data - Examining Relationships

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Mathematics Clarification Document
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Mathematics

Clarification for Standard S2: Bivariate Data – Examining Relationships

Strand: S-Statistics

In Kindergarten through Grade 8, students develop the ability to read, analyze, and construct a repertoire of statistical graphs. Students also examine the fundamentals of experimental and theoretical probability in informal ways. The Basic Counting Principle and tree diagrams serve as tools to solve simple counting problems in these grades.

During high school, students build on that foundation. They develop the data interpretation and decision-making skills that will serve them in their further study of mathematics as well as in their coursework in the physical, biological, and social sciences. Students learn important skills related to the collection, display, and interpretation of both univariate and bivariate data. They understand basic sampling methods and apply principles of effective data analysis and data presentation. These skills are also highly valuable outside of school, both in the workplace and in day-to-day life.

In probability, students utilize probability models to calculate probabilities and make decisions. The normal distribution and its properties are studied. Students then use their understanding of probability to make decisions, solve problems, and determine whether or not statements about probabilities of events are reasonable. Students use technology when appropriate, including spreadsheets. This strong background in statistics and probability will enable students to be savvy decision-makers and smart information-consumers and producers who have a full range of tools in order to make wise choices.

Standard: S2 - Bivariate Data – Examining Relationships

Students plot and interpret bivariate data by constructing scatterplots, recognizing linear and nonlinear patterns, and interpreting correlation coefficients; they fit and interpret regression models, using technology as appropriate.

Topic S2.1: Scatterplots and Correlation

HSCE: S2.1.1 Construct a scatterplot for a bivariate data set with appropriate labels and scales.

Clarification: Continuation of work done in 7th grade

HSCE: S2.1.2 Given a scatterplot, identify patterns, clusters, and outliers; recognize no correlation, weak correlation, and strong correlation.

Clarification: Students should be able to recognize when the pattern suggests a linear relationship between the two variables, to recognize when a lurking variable may be present, to recognize points that are outliers, and to recognize that the scales on the horizontal and vertical axes can affect how strong the correlation seems to be.

HSCE: S2.1.3 Estimate and interpret Pearson's correlation coefficient for a scatterplot of a bivariate data set; recognize that correlation measures the strength of linear association.

Clarification: Given a scatterplot, students should be able to give a rough estimate of the correlation by eye. Students should also recognize plots that show a strong (nonlinear) association but where the correlation will be close to zero.

HSCE: S2.1.4 Differentiate between correlation and causation; know that a strong correlation does not imply a cause-and-effect relationship; recognize the role of lurking variables in correlation.

Clarification: Students should be able to recognize and construct examples where there is a strong correlation but where a causal relationship is dubious (or clearly untrue). In particular students should be able to point to the role of lurking variables in potentially pointing to a causal relationship when none exists.

Topic S2.2: Linear Regression

HSCE: S2.2.1 For bivariate data which appear to form a linear pattern, find the least squares regression line by estimating visually and by calculating the equation of the regression line; interpret the slope of the equation for a regression line.

Clarification: Much of this is already included in the Grade 7 GLCEs. The principle of "least squares" which underlies the regression line (called the "line of best fit" in the Grade 7 GLCEs) is new, important, and should be stressed. In addition, students should be able to easily interpret the slope of the regression line in a variety of contexts.

HSCE: S2.2.2 Use the equation of the least squares regression line to make appropriate predictions.

Clarification: Some of this is already included in the Grade 7 GLCEs. Here the students' understanding should be more sophisticated, and should include an understanding that the predictions are more reliable near the "center" of the data, and that predictions that involve extrapolation are potentially inaccurate. Students should be able to recognize and construct examples where the linear relationship changes outside the range of the data, and where extrapolation is unwise.

Background Information, Tools, and Representations

- It is important that students understand that the correlation coefficient tells the strength of the linear relationship – it is calculated solely from the data values; it does not verify the fit of a particular linear model. There is however a technological difficulty; most graphing calculators will not display the correlation coefficient without showing the least squares regression line.

Assessable Content

Resources

Annenberg Video Library, Video 7: Bivariate Data and Analysis
Analyze bivariate data and understand the concepts of association and co-variation between two quantitative variables. Explore scatter plots, the least squares line, and modeling linear relationships.
http://www.learner.org/resources/series158.html

Clarifying Examples and Activities

HSCE: S2.1.1 Construct a scatterplot for a bivariate data set with appropriate labels and scales.

HSCE: S2.1.2 Given a scatterplot, identify patterns, clusters, and outliers; recognize no correlation, weak correlation, and strong correlation.

HSCE: S2.1.3 Estimate and interpret Pearson's correlation coefficient for a scatterplot of a bivariate data set; recognize that correlation measures the strength of linear association.

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HSCE: S2.2.2 Use the equation of the least squares regression line to make appropriate predictions.

Example 1:

On his bicycle, Mr. Schnepp has a computer / speedometer that tells him various kinds of information about his riding. Three data functions on the computer are "cadence" (pedaling rate in rpm), "bike speed" (mph), and "heart rate" in (bpm).

As Mr. Schnepp rides for a period of time in a particular gear, with a particular cadence, the bike travels at a certain speed and his heart beats at a particular rate.

Pedaling Rate Input (rpm)	60	79	113	121	53
Heart Rate (bpm)	162	171	180	181	178
Bike Speed (mph)	18.6	24.2	34.5	37.4	16.2

Make one scatterplot with the variables pedaling rate and bike speed.
Make one scatterplot with the variables pedaling rate and heart rate.
Make one scatterplot with the variables heart rate and bike speed.
Describe any patterns you see in the data sets.
Based on your interpretation are linear models appropriate for any of these paired data sets? Explain.
Of the relationships that you have graphed, are they all cause and effect relationships?
Find and interpret the correlation coefficient for each paired data set.

Example 2

Using the data from Example 1, explore the following:

Choose two points that represent the trend in the data using the variables pedaling rate and bike speed. Use your two points to write a rule for a linear model for this relationship. Give the slope with correct units and explain what this number tells you about Mr. Schnepp on his bike.

Choose two points that represent the trend in the data using the variables pedaling rate and heart rate. Use your two points to write a rule for a linear model for this relationship. Give the slope with correct units and explain what this number tells you about Mr. Schnepp on his bike.

Choose two points that represent the trend in the data using the variables heart rate and pedaling rate. Use your two points to write a rule for a linear model for this relationship. Give the slope with correct units and explain what this number tells you about Mr. Schnepp on his bike.

If Mr. Schnepp pedals at 100 rpm, calculate predictions of his heart rate and bike speed.

If Mr. Schnepp's bike is traveling at 45 mph, calculate predictions for heart rate and pedaling rate.

Repeat the entire process using graphing calculator / other technology to find the least squares regression line and scatter plots. Compare your linear model with the regression model generated by the calculator.

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